

Resistive and propulsive forces in swimming: challenges and novel insights

icSPORTS 2025

Paola Zamparo



UNIVERSITÀ
di **VERONA**

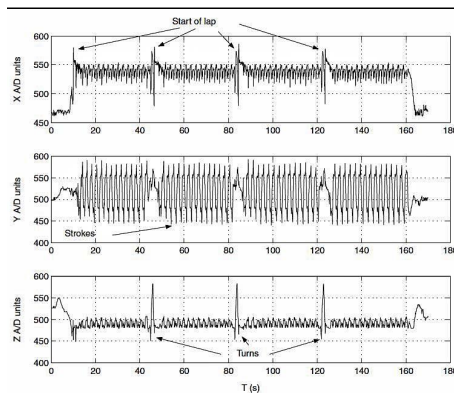
Propulsive and resistive forces in water locomotion

Constant speed

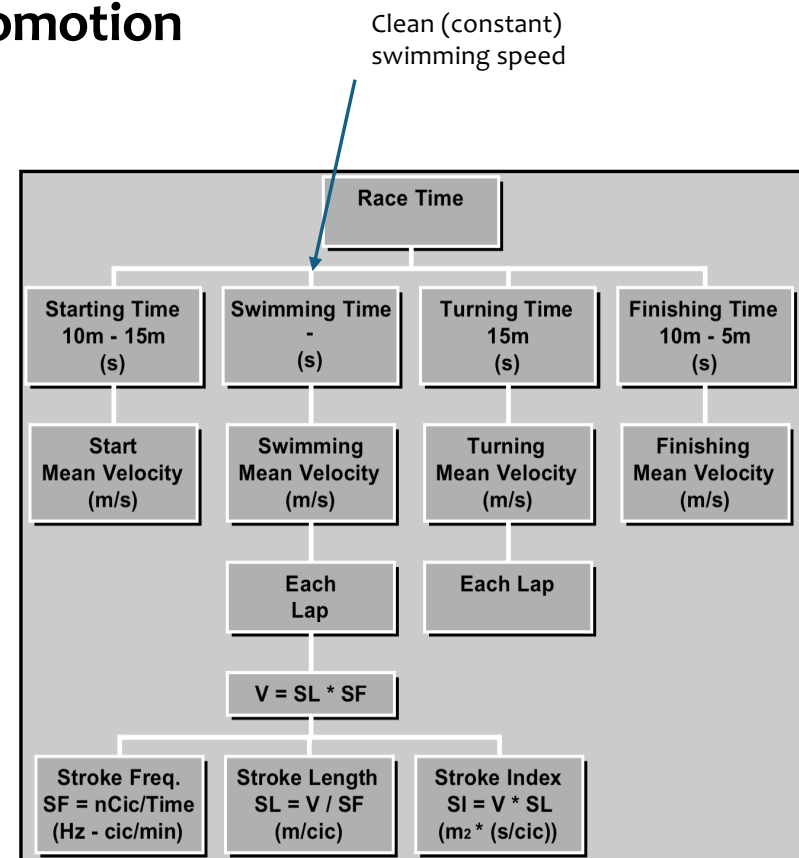
$$F_P + F_{D,a} = 0$$

$$F_P = -F_{D,a}$$

F_P = propulsive force
 $F_{D,a}$ = hydrodynamic resistance (active drag)

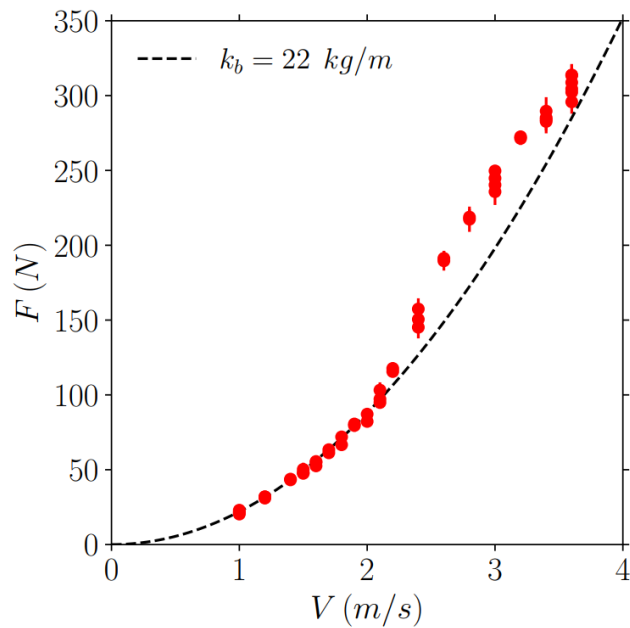


Accelerometer data
 (200 m race, 25 m pool)



Hay 1993

Resistive forces: passive drag

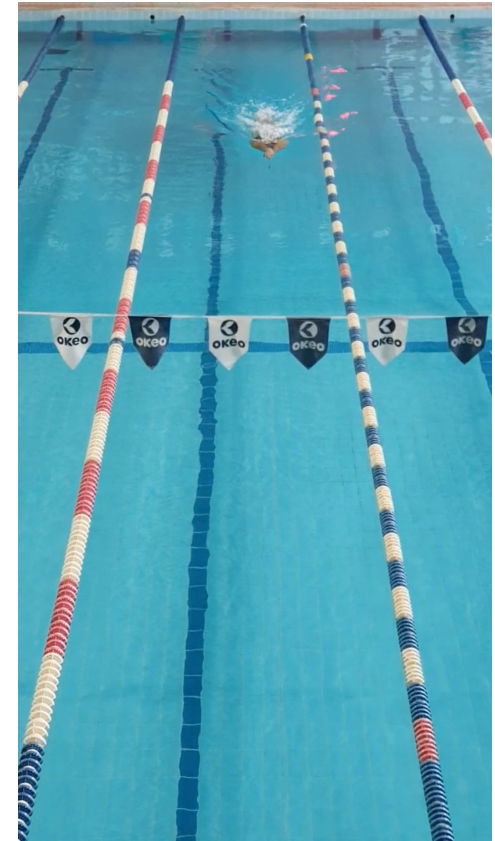


$$F_{D,p} = k_p v^2$$

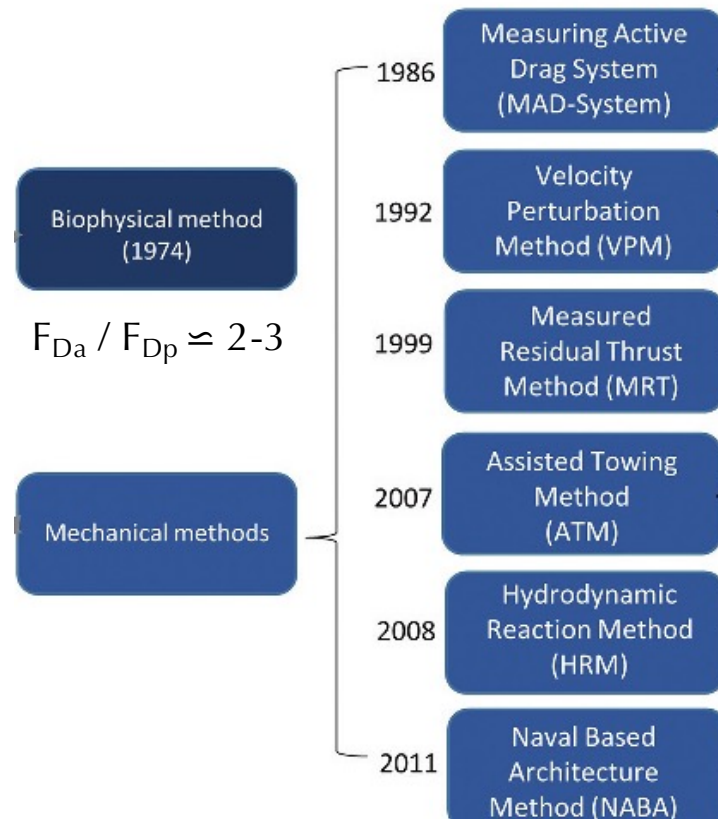
$$k_p = \frac{F_{D,p}}{v^2}$$

Passive drag values show a remarkable consistency across different experimental designs (Havriluk 2005)

Pretot et al. (2022)



Resistive forces: active drag



Sacilotto et al. (2023)

active to passive drag ratio

$F_{Da} / F_{Dp} \approx 1-1.2$

$F_{Da} / F_{Dp} \approx 1, \text{ or less}$

$F_{Da} / F_{Dp} \approx 2$

$F_{Da} / F_{Dp} \approx 2$

$F_{Da} / F_{Dp} \approx 2$

The variability of the active drag values reflects the difficulty in measuring active drag and the imaginative attempts that have been designed to analyze the complex activity of human swimming (Havriluk 2005)

Resistive forces: active drag

Methods based on perturbations

Equal thrust assumption

- MRT method (and NABA method)
- Biophysical method

Equal power assumption

- Velocity perturbation method
- Assisted towing method

Other methods

- Planimetric method
- MAD system

The MAD system ($F_{Da} / F_{Dp} \approx 1-1.2$)

Toussaint et al. (1988)

Assumption: at constant speed
the mean force measured by
the underwater pads equals the
swimmer's drag force

Assumption: when pushing on
pads no energy is wasted in
giving water kinetic energy and
propelling efficiency = 100%



No leg action is allowed (the swimmer uses a pulley buoy):
this system measures the active drag of the arm stroke only.

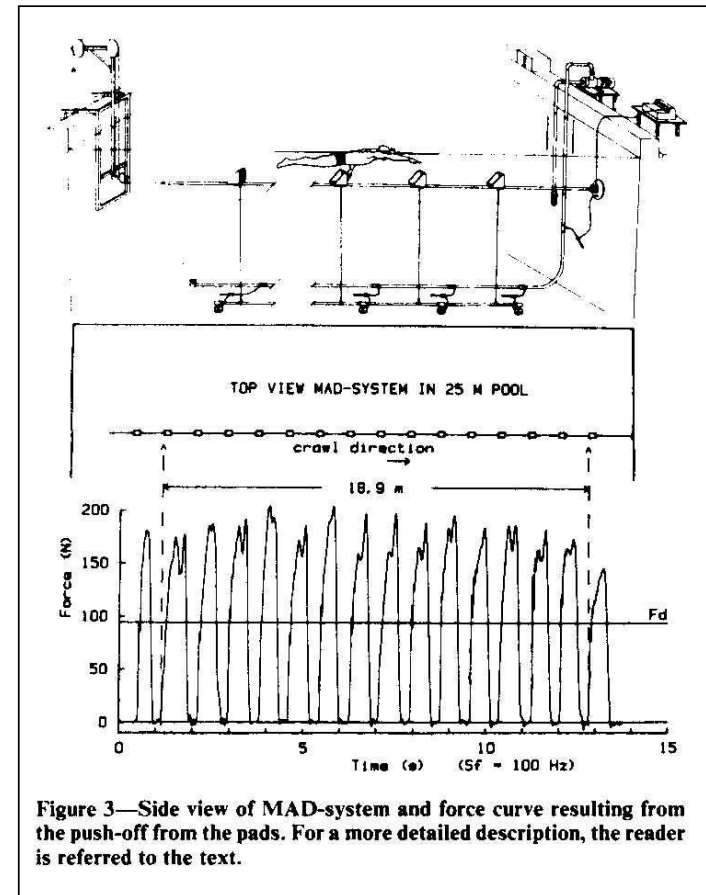
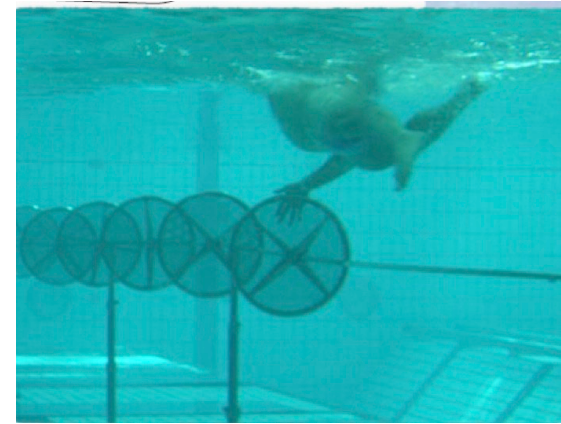


Figure 3—Side view of MAD-system and force curve resulting from the push-off from the pads. For a more detailed description, the reader is referred to the text.

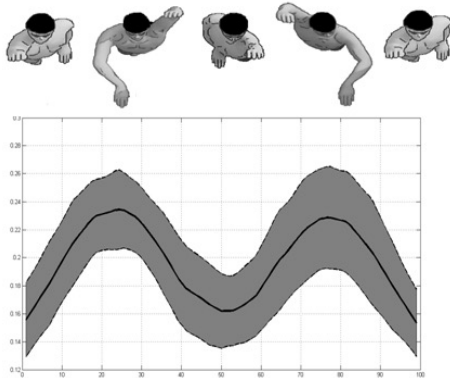
The MAD system ($F_{Da} / F_{Dp} \approx 1-1.2$)

Havriluk (2005): this method underestimate active drag because the pads:

- only measure hand force and do not include any propulsive force generated by the legs
- only measure force when the hand is in contact with a pad and not the entire time the hand is submerged
- do not permit the hand to move laterally, vertically or horizontally as during actual swimming
- The legs are floated by a pull buoy: the swimmer is more horizontal in water than it would be without the pull buoy (lower frontal area than in actual swimming conditions)



The planimetric method ($F_{Da} / F_{Dp} \approx 1.5$)

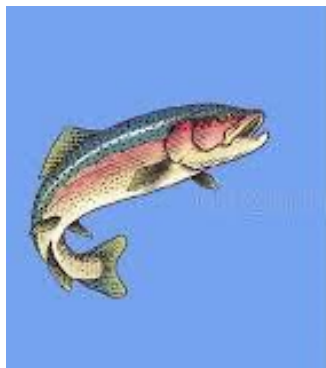


Gatta et al. (2015)

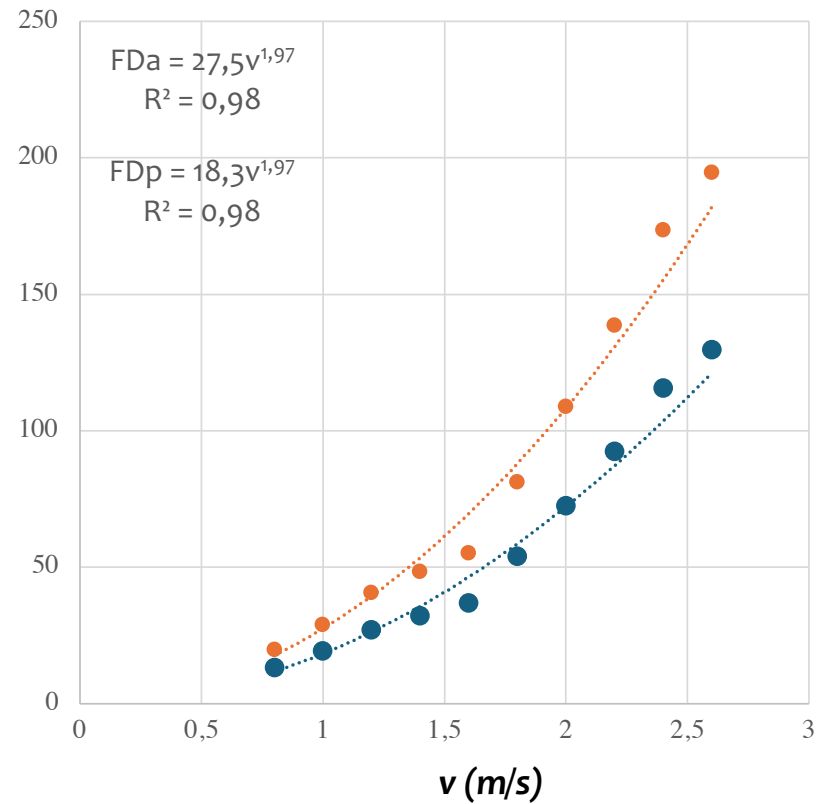
Assumption: differences between passive and active drag depend (essentially) on differences in frontal area

$$F_D = \frac{1}{2} \rho A C_d v^2$$

In front crawl:
 $A_a = 1.5 A_p$
 $F_{Da} = 1.5 F_{Dp}$
 (average over one cycle)

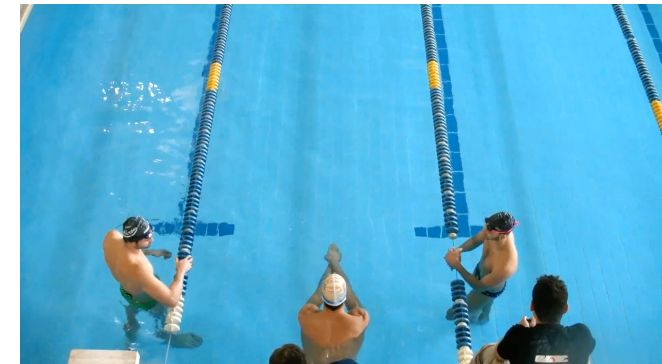
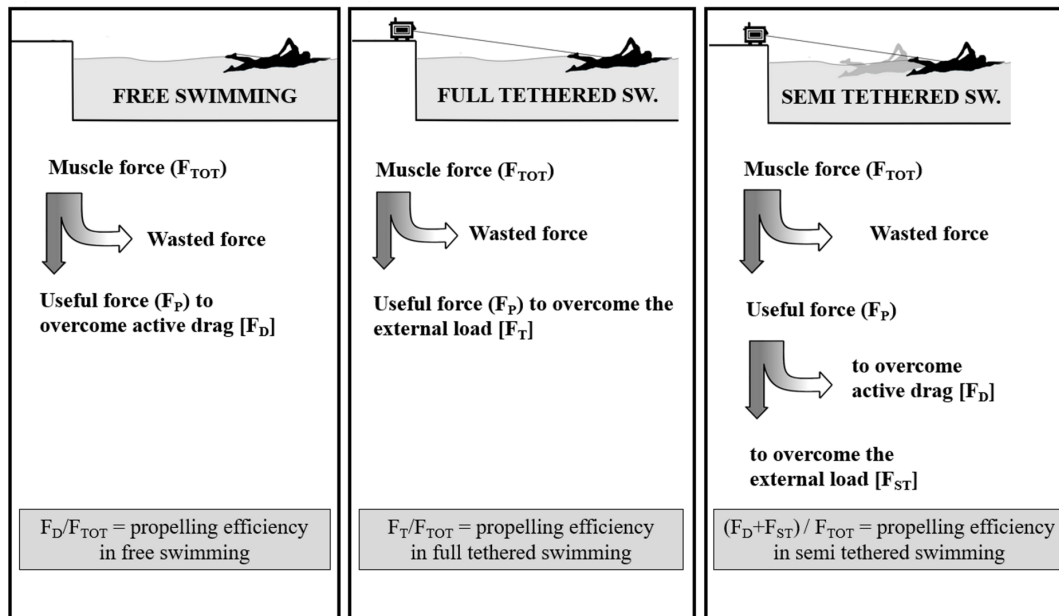


$D(N)$



The measured residual thrust method (MRT, $F_{Da} / F_{Dp} \approx 2$)

Full and semi-tethered tests - Equal thrust assumption



Takagi et al. (1999) Narita et al. (2017) Cortesi et al. (2024)

Assumptions:

Equal thrust ($F_P = \text{costant}$)

$$F_{D,a} = k_a v^2 \quad (k_a = \text{costant})$$

Full tethered swimming

$$F_{P,T} = F_T$$

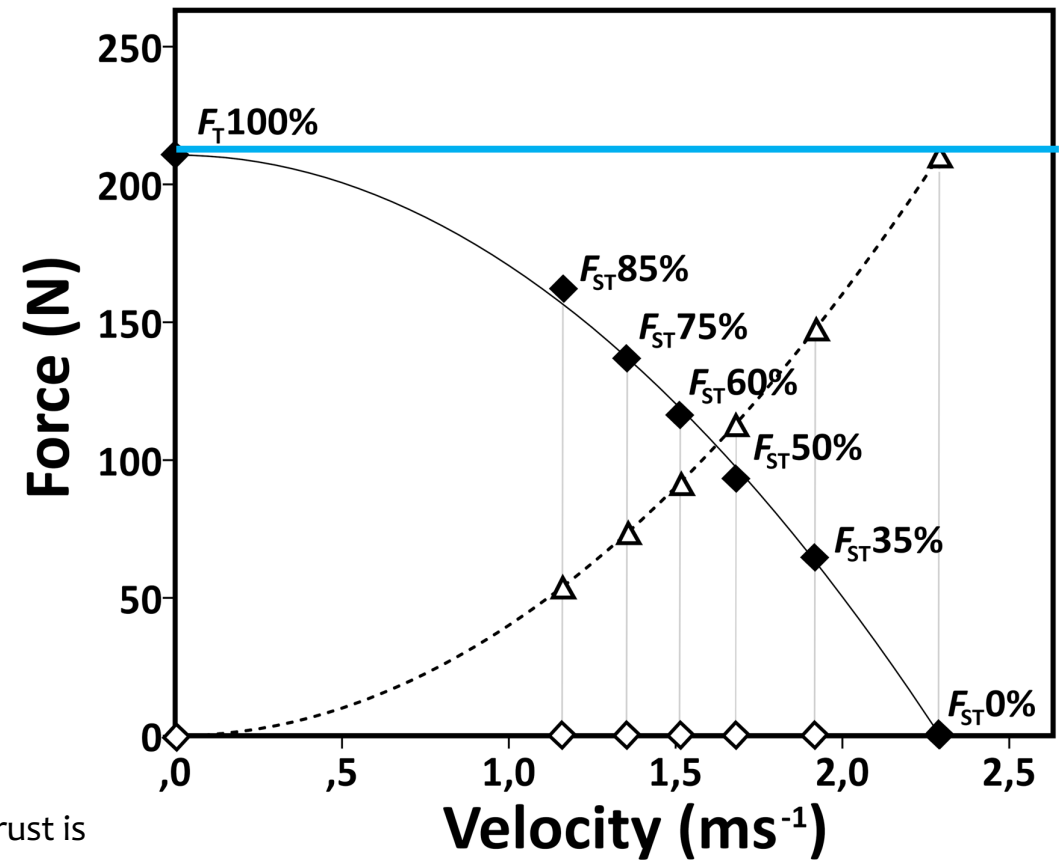
Semi tethered swimming

$$F_{P,ST} = F_{ST} + k_{a,ST} v_{ST}^2$$

$$F_{P,ST} - F_{ST} = k_{a,ST} v_{ST}^2$$

$$k_{A,ST} = (F_P - F_{ST}) / v_{ST}^2$$

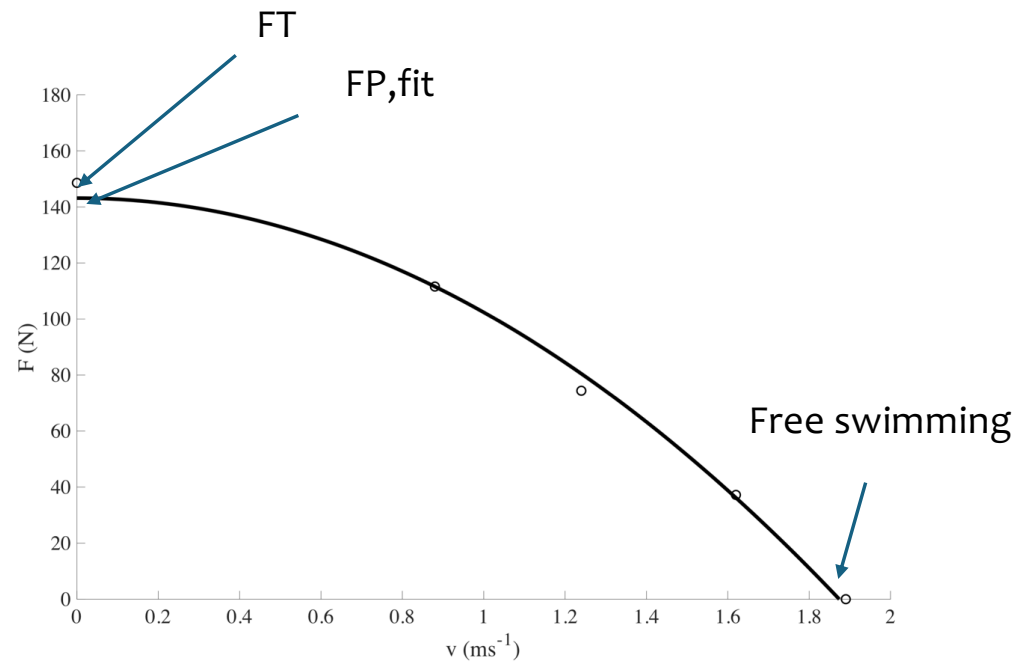
The residual thrust is
the active drag



Carmignani et al. (2025)

Cortesi et al. (2024)

The measured residual thrust method (MRT, $F_{Da} / F_{Dp} \approx 2$)



Input output input

$$F_{ST} = F_P - k_A v_{ST}^2$$

$$F_{P, fit} = 144 \text{ N}$$

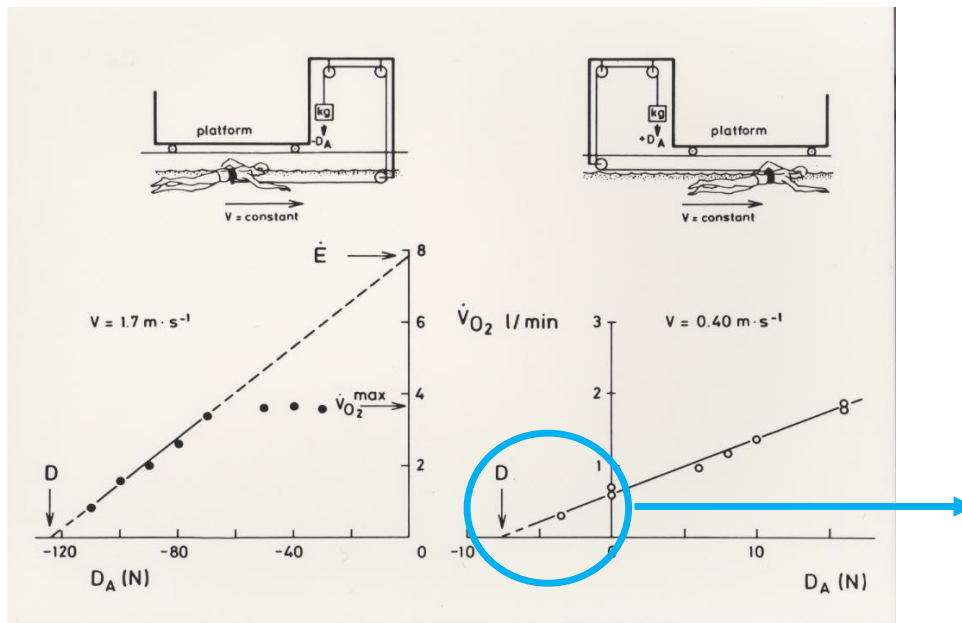
$$ka_{ST, fit} = 41 \text{ Kg/m}$$

Cortesi et al. (2024)

Carmignani et al. (2025)

The Biophysical method ($F_{Da} / F_{Dp} = 2-3$)

Equal thrust assumption

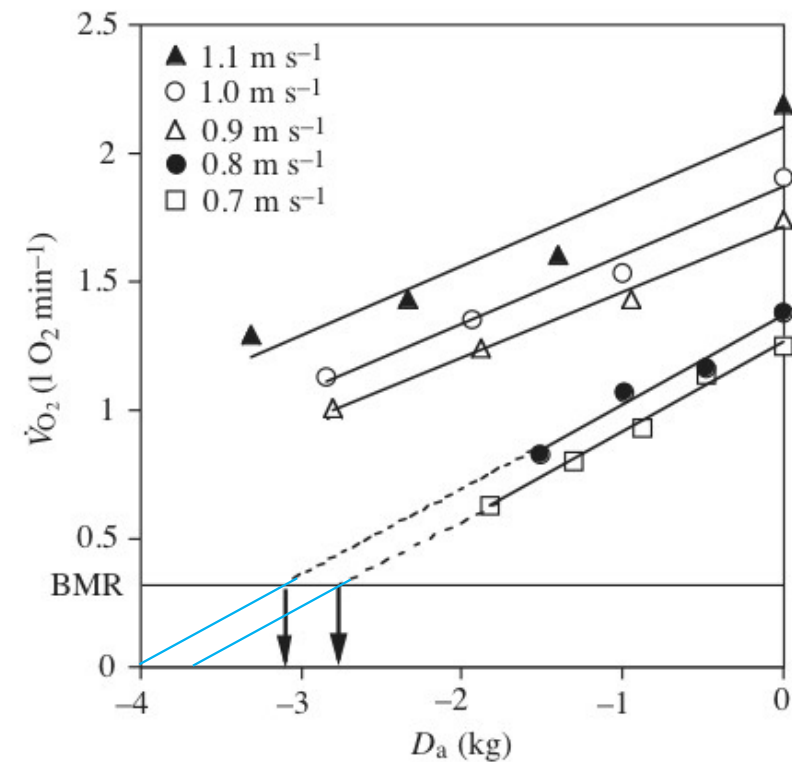


di Prampero and Pendergast (1974)

($F_{Da} / F_{Dp} \approx 3$)

Zamparo et al. (2003)

($F_{Da} / F_{Dp} \approx 2$)



The velocity perturbation method ($F_{Da} / F_{Dp} \approx 1$)

Equal power assumption

$$P_1 = P_2$$

$$F_1 = \frac{F_b \cdot V_2 \cdot V_1^2}{V_2^3 - V_1^3}$$

$$P_1 = F_1 \times v_1$$

F_1 = active drag

v_1 = free swimming speed (max)

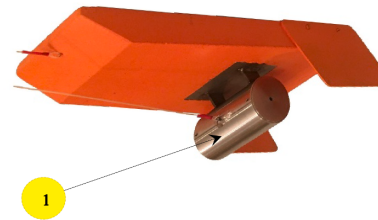
$$P_2 = F_2 \times v_2$$

v_2 = swim speed with AHB (max)

$$F_2 = F_1 + F_b$$

F_b = passive drag of AHB

AHB: Added hydrodynamic body



The equal power assumption
is easily violated

**V_1 and V_2 should not differ
more than 10%**

Kolmogorov and Duplishcheva (1992)
Kolmogorov (2023)

The assisted towing method ($F_{Da} / F_{Dp} \approx 2$)

The equal power assumption: $P_1 = P_2$

$$F_1 = \frac{F_b \cdot V_2 \cdot V_1^2}{V_2^3 - V_1^3}$$

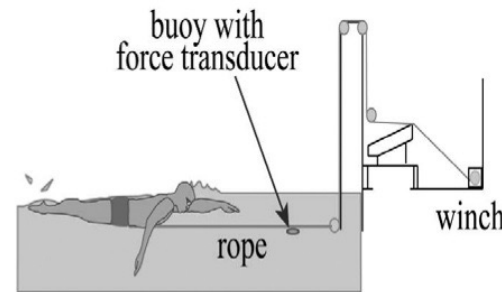


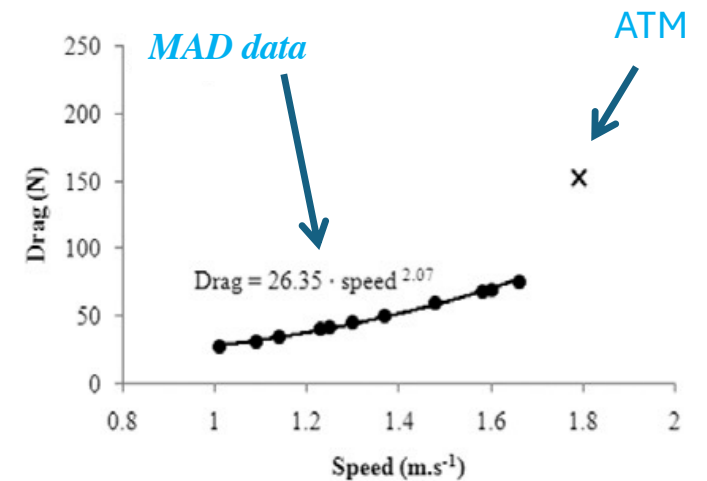
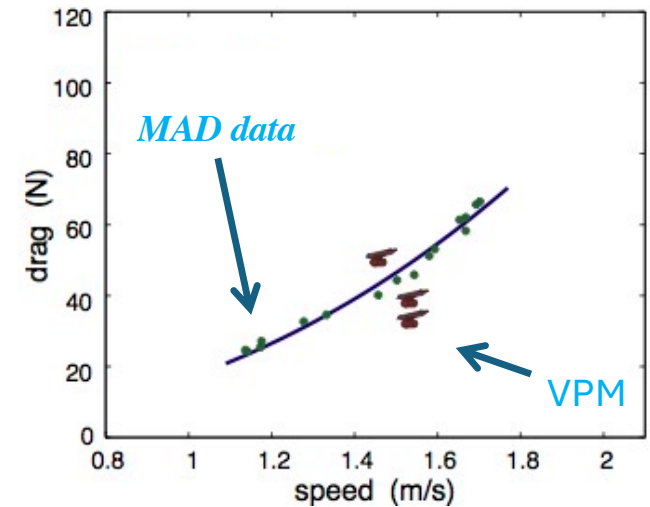
Figure 3 — Schematic drawing of the ATM mounted onto a starting block.

V_1 and F_1 : free swimming

V_2 and F_2 : assisted towing

F_b = towing force (at $v > 10\%$ of v_{max})

Formosa et al. (2012)



Toussaint, Roos, Kolmogorov (2004)

Methods	F_{Da} / F_{Dp}	Materials and data analysis	Outcomes
MAD system	$\cong 1-1.2$	<i>Easy to use; MAD system</i>	front crawl (arm stroke) range of speeds (F vs. v)
VPM method	$\cong 1$	<i>Easy to use; very cheap</i>	all strokes maximal speed only
ATM method	$\cong 2$	<i>Easy to use; force transducer</i>	front crawl maximal speed only
Planimetric method	$\cong 1.5$	<i>Easy to use; force transducer (Dp) The protocol is for research only</i>	all strokes range of speeds (F vs. v)
Residual thrust method	$\cong 2$	<i>The protocol is for research only and it is time consuming</i>	front crawl range of speeds (F vs. v)
Biophysical method	$\cong 2$	<i>The protocol is for research only and it is time consuming Metabolic data should also be collected</i>	front crawl (breaststroke) range of aerobic speeds (F vs. v)

The variability of the active drag values reflects the difficulty in measuring active drag and the imaginative attempts that have been designed to analyze the complex activity of human swimming (Havriluk 2005)

Propulsive forces (in water)

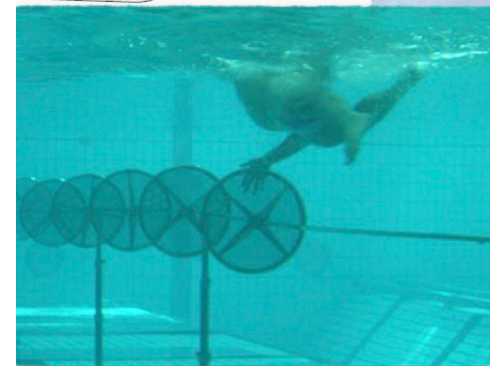
The forces are actually measured

- MAD system
- Full and semi-tethered tests

Other measures

- Hand paddles (pressure sensors)
- Inertial sensors
- Kinematic data

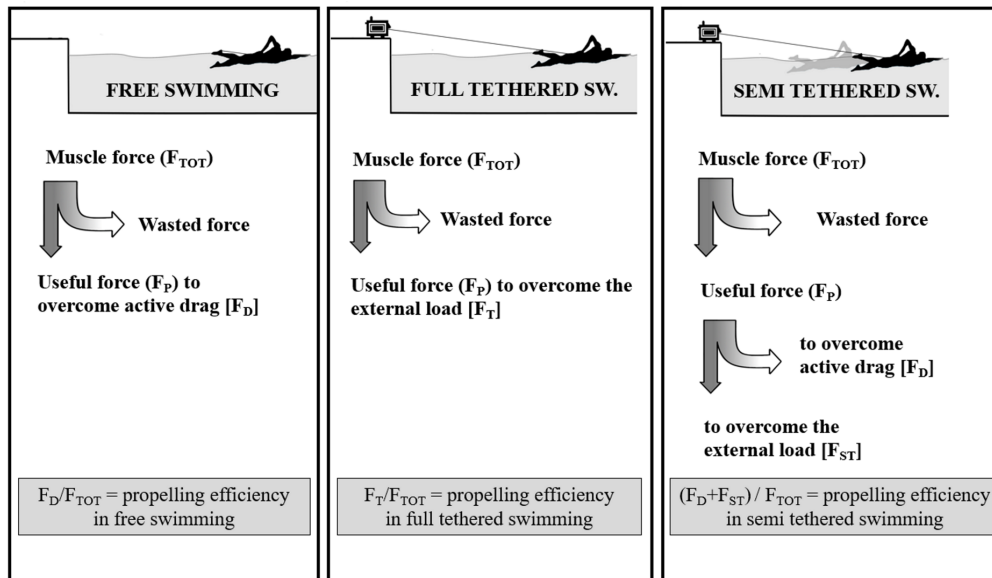
MAD system



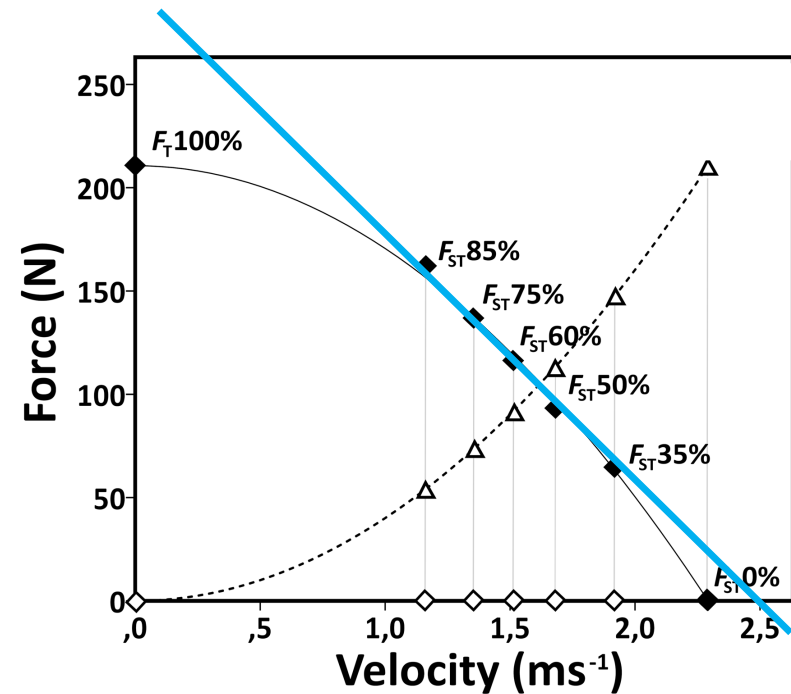
*Assumption: at constant speed
the mean force measured by
the underwater pads equals the
swimmer's drag force*

*Assumption: when pushing on
pads no energy is wasted in
giving water kinetic energy and
propelling efficiency = 100%
muscle force = useful force*

Full and semi-tethered tests



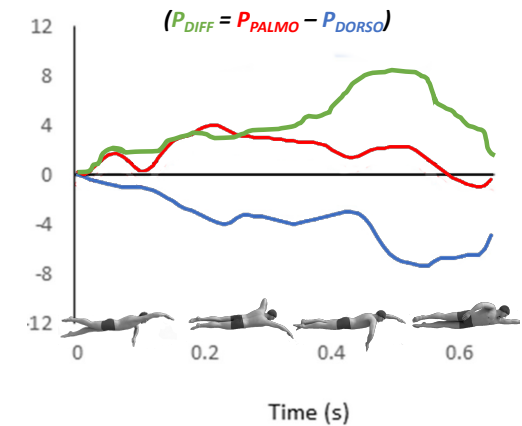
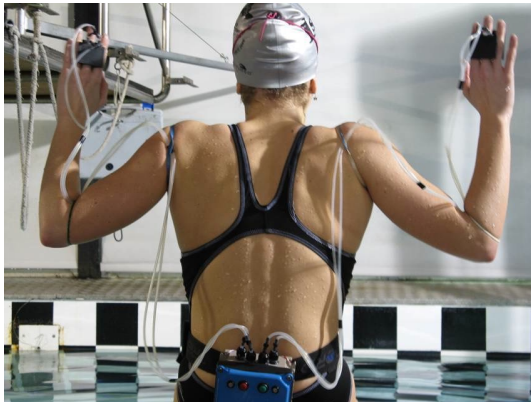
propelling efficiency < 100%
muscle force does not correspond to the useful force



Other measures: Hand paddles

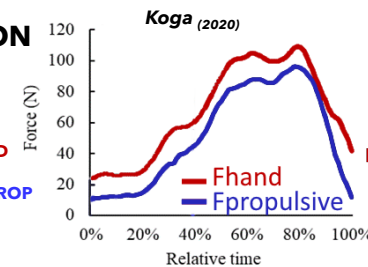
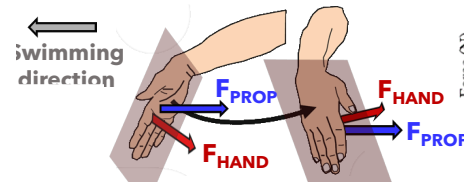
$$P = F/A$$

Force = pressure x area

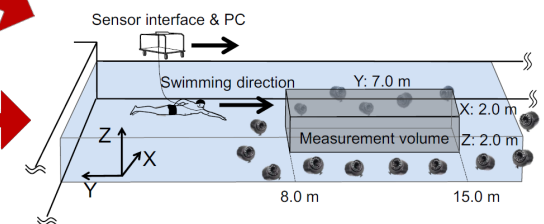


FOREARM CONTRIBUTION:
25% of upper-arm contribution

NO HAND PLANE ORIENTATION

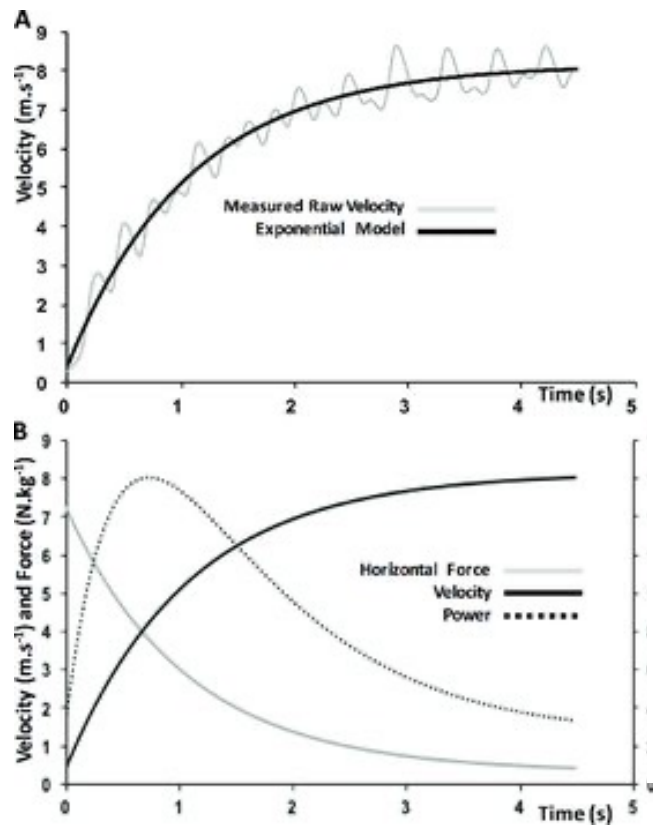


Kinematic Analysis

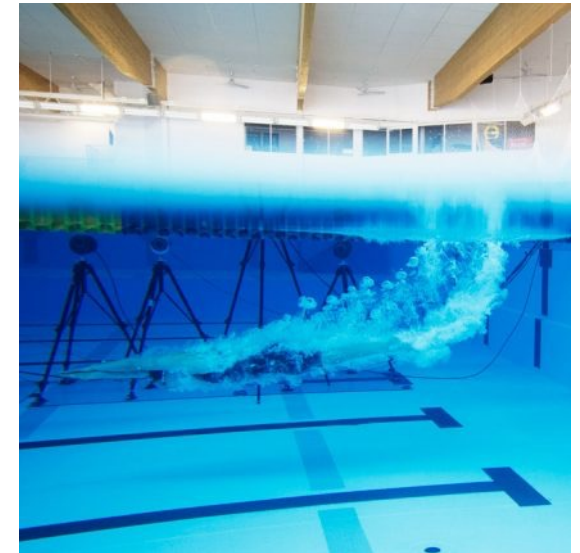
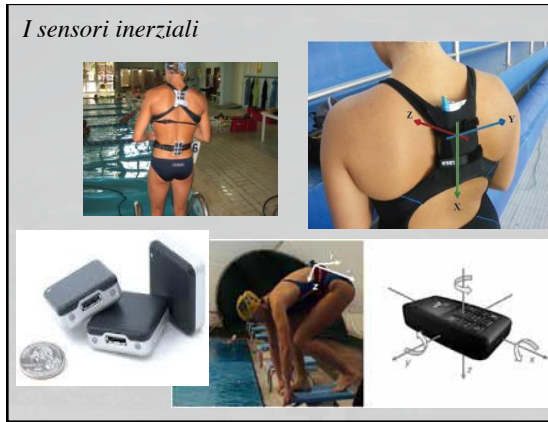


Other measures: inertial sensors and kinematic analysis

Sprint running



I sensori inerziali



$$F = ma$$

At (mean) constant speed $a = 0$

During starts and turns $a \neq 0$

the added mass should be taken into consideration

The standing start test

(to measure/estimate propulsive force and hydrodynamic resistance during a sprint)

Constant speed

$$F_P + F_{D,a} = 0$$

$$F_P = -F_{D,a}$$

F_P = propulsive force
 $F_{D,a}$ = hydrodynamic resistance (active drag)
 M_0 = swimmer's body mass
 $M_{A,a}$ = added mass (active conditions)
 v = speed
 t = time
 a = acceleration

Unsteady speed

$$F_P + F_{D,a} + F_{in} = 0$$

$$F_P = -(F_{D,a} + F_{in})$$

$$F_{in} = (M_0 + M_{A,a}) a$$

$$F_P = (F_{D,a} + (M_0 + M_{A,a})) a$$

$$F_P = (F_{D,a} + (M_0 + M_{A,a})) \frac{dv}{dt}$$

During a sprint, besides the drag force, the swimmer experiences an additional (inertial) force: some of the water around the swimmer is set in motion and this can be thought of as an added mass the swimmer has to accelerate, in addition to body mass

Added mass (passive)

The natural frequency of a damped spring system (ω_0) depends on the spring constant (k) and on the total mass in motion (M_A = added mass; M = mass of the swimmer)

$$\omega_0 = \sqrt{\frac{k}{M + M_A}}$$

$$T = \frac{2\pi}{\omega_0}$$

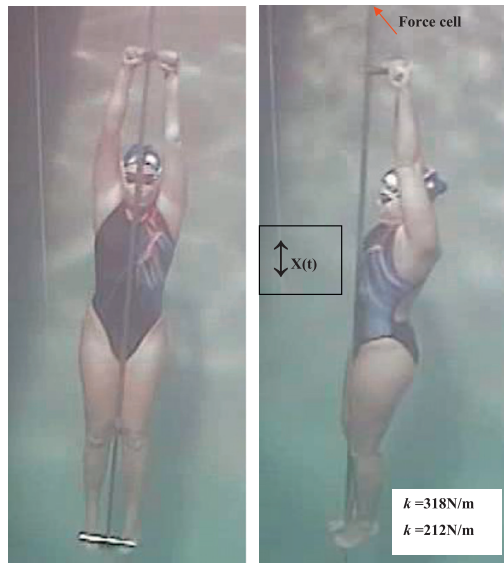


Fig. 1. A swimmer attached to the bar during vertical oscillations.

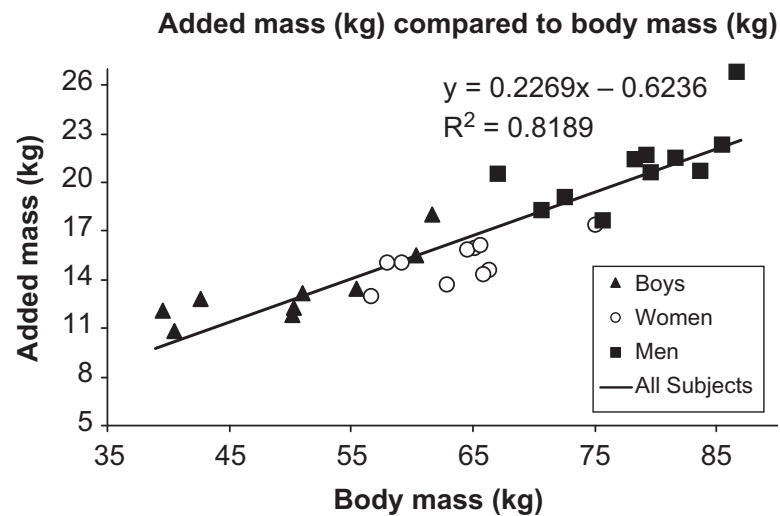


Fig. 3. Added mass versus body mass for boys (▲) women (○) and men (■).

Added mass coefficient
 ≈ 0.25

Added mass = $0.25 \cdot 70 \text{ kg}$
 $\approx 17.5 \text{ kg}$

Caspersen et al. (2010)

Added mass (passive, kayaking)

The inertia of the kayaker and his boat depends on the total mass (M_{tot})

M_k = the mass of the kayaker (70 kg)

M_b = the mass of the boat (12 kg)

M_a = the added mass

The *added mass coefficient* could be estimated based on the aspect ratio of the kayak
(a/b where a = length and b = width)

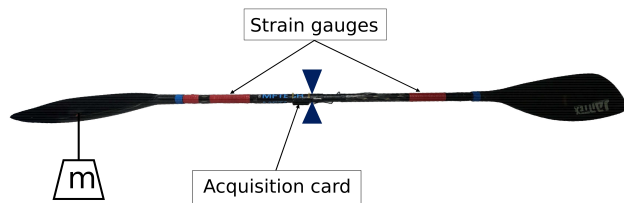
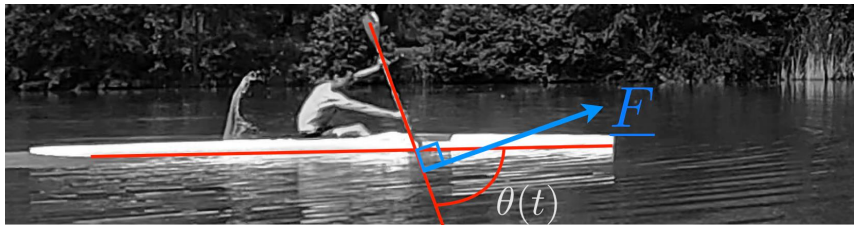
Added mass coefficient
(M_a/M_{tot})
 ≈ 0.017

Added mass = $0.017 \cdot 82 \text{ kg}$
 $\approx 1.4 \text{ kg}$



Pretot et al. (2022)

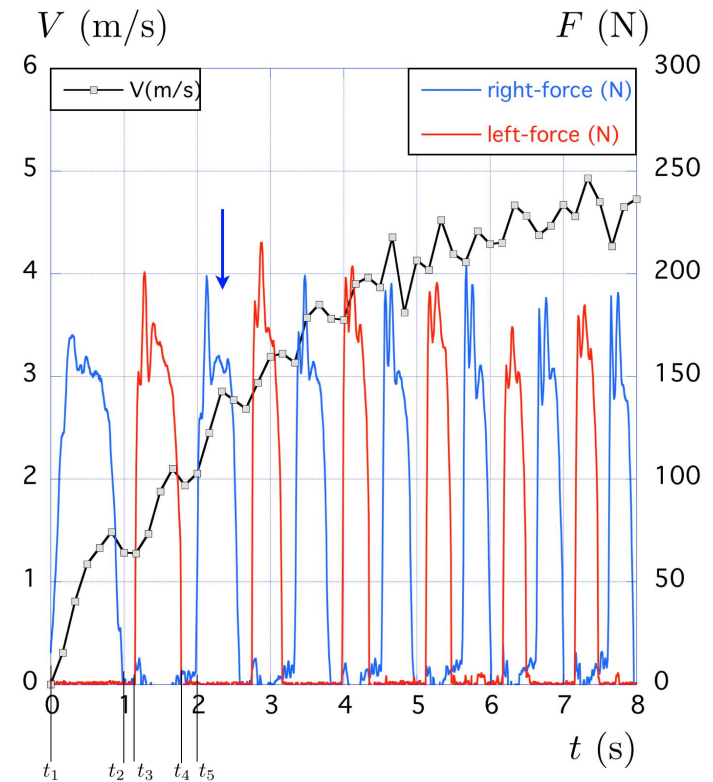
Standing start test (kayaking)



$$F_P = F_{D,a} + (M_0 + M_{A,a}) \frac{dv}{dt}$$

the unknown

$$v(t) = v_{\max} \tanh\left(\frac{t}{\tau}\right)$$



Propulsive force is “constant” in the
acceleration phase of a sprint
(Equal thrust assumption)

Pretot et al. (2022)

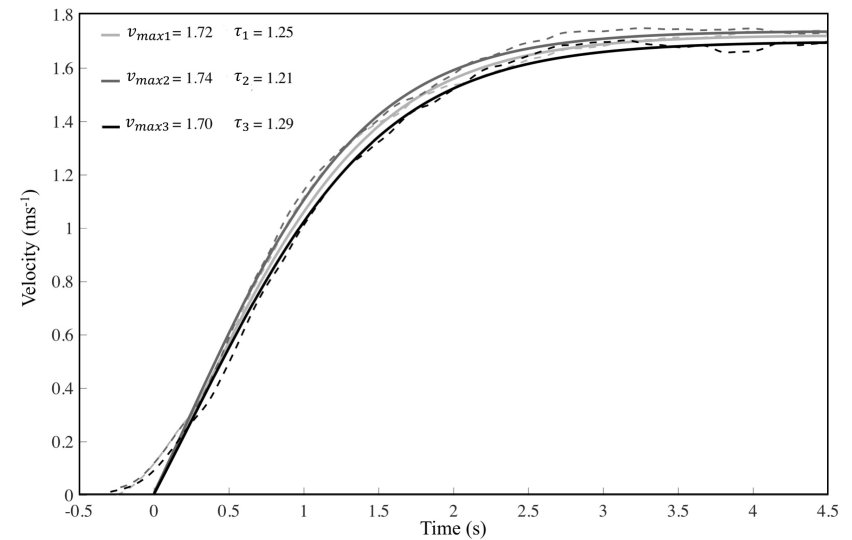
Standing start test (swimming)



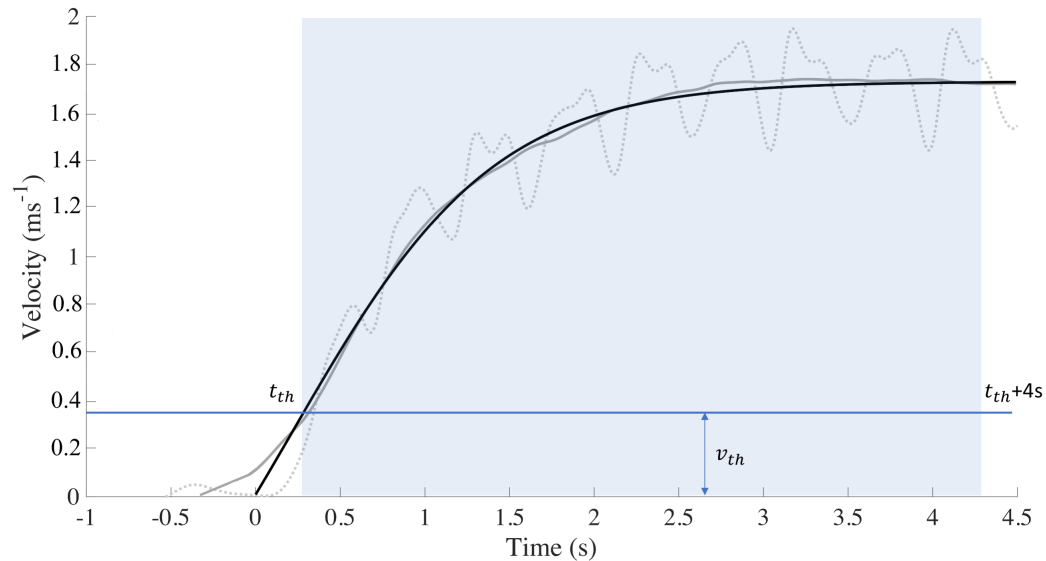
$$F_P = F_{D,a} + (M_0 + M_{A,a}) \frac{dv}{dt}$$

Standing start method

$$v(t) = v_{\max} \tanh\left(\frac{t}{\tau}\right)$$



Carmignani et al. (2025)



a threshold velocity (v_{th}) is chosen so to eliminate the contribution of the first part of the trial (when the velocity increase is less steep) and, thus, minimize the uncertainty in the estimation of τ

T is the mean cycle duration calculated from consecutive wrist entries

$$v(t_i) = \frac{1}{T} \int_{t_i - \frac{T}{2}}^{t_i + \frac{T}{2}} V(t) dt$$

$$v(t) = v_{\max} \tanh\left(\frac{t - t_0}{\tau}\right)$$

$v_{\max}$ corresponds to the maximal velocity, τ is the characteristic acceleration time and t_0 is the estimated start of the swimmer's acceleration

$$F_P = F_{D,a} + (M_0 + M_{A,a}) \frac{dv}{dt}$$

$$F_P = k_a v^2 + (M_0 + M_{A,a}) \frac{dv}{dt}$$

$$v(t) = \sqrt{F_P/k_a} \tanh\left(\frac{t}{(M_0 + M_{A,a})/\sqrt{k_a F_P}}\right)$$

$$v(t) = v_{max} \tanh\left(\frac{t}{\tau}\right)$$

$$v_{max} = \sqrt{F_P/k_a} \quad \tau = \frac{(M_0 + M_{A,a})}{\sqrt{k_a F_P}}$$

Assumptions:

$F_P = \text{constant}$

$F_{D,a} = kv^2$

F_P = propulsive force

$F_{D,a}$ = hydrodynamic resistance (active drag)

k_a = speed - specific drag (active conditions)

M_0 = swimmer's body mass

$M_{A,a}$ = added mass (active conditions)

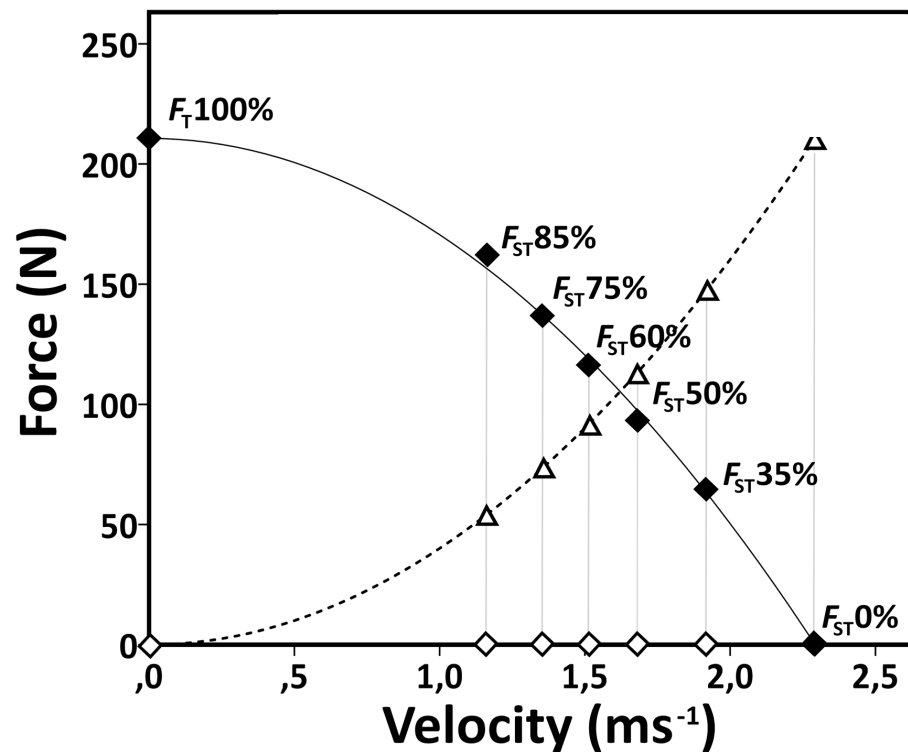
v = speed

t = time

a = acceleration

$$v_{max} \tau = \frac{(M_0 + M_{A,a})}{k_a}$$

$$\frac{v_{max}}{\tau} = \frac{F_P}{(M_0 + M_{A,a})}$$



Cortesi et al. (2024)

Carmignani et al. (2025)

speed-specific drag in active conditions can be calculated by the fitting of the experimental data

Input output input

$$F_{ST} = F_P - k_A v_{ST}^2$$

$k_{A,ST, fit}$ $F_{P, fit}$

speed-specific drag in active conditions can be calculated for each pair of force/speed values.

$$k_{A,ST} = (F_P - F_{ST}) / v_{ST}^2$$

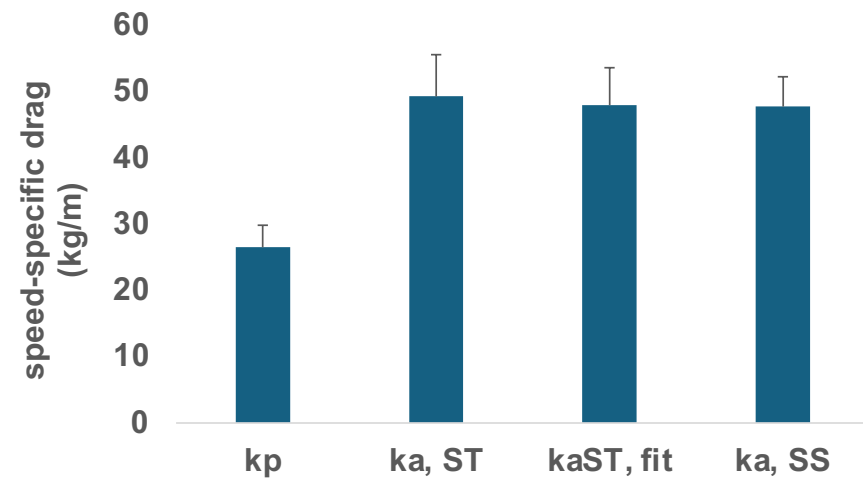
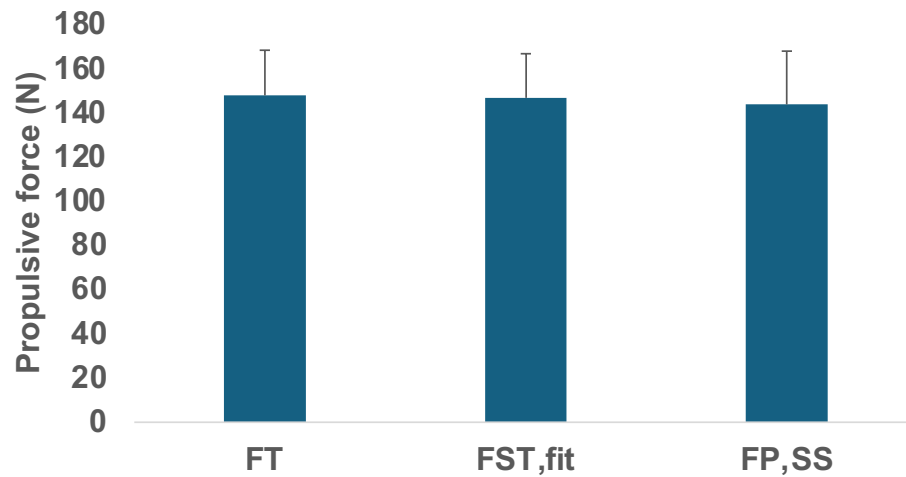
$$F_{P,T} = F_T$$

$$v_{max} \tau = \frac{(M_0 + M_A)}{k_a}$$

$$\frac{v_{max}}{\tau} = \frac{F_P}{(M_0 + M_A)}$$

With a standing start test it is possible to estimate the propulsive force and the active drag coefficient of a swimmer (based on values of passive/active added mass)

25% body mass
(Caspersen et al. 2010)



Passive drag

Carmignani et al. (2025)

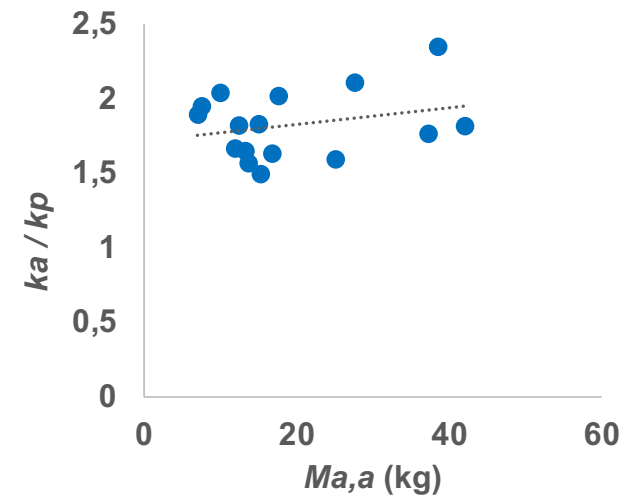
$$v_{max} \tau = \frac{(M_0 + M_{A,a})}{k_a}$$

$$\frac{v_{max}}{\tau} = \frac{F_P}{(M_0 + M_{A,a})}$$

With a standing start test it is possible to estimate the (active) added mass during swimming and at the surface (based on values of propulsive force or of active drag coefficient)

$M_{A,a} / M_0$

	Added mass coefficient	Added mass Range
Caspersen et al. (2010)	$25 \pm 3\%$	15 - 21 kg
$F_P = F_T$	$30 \pm 15\%$	11 - 43 kg
$F_P = F_{P,ST \text{ fit}}$	$29 \pm 15\%$	8 - 44 kg
$k_a = k_{a,ST \text{ fit}}$	$26 \pm 15\%$	7 - 42 kg
$k_a = k_{a,ST}$	$30 \pm 17\%$	6 - 53 kg



Carmignani et al. (2025)

Methods	F_{Da} / F_{Dp}	Materials and data analysis	Outcomes
MAD system	$\approx 1-1.2$	Easy to use; MAD system	front crawl (arm stroke) range of speeds (F vs. v)
VPM method	≈ 1	Easy to use; very cheap	all strokes maximal speed only
ATM method	≈ 2	Easy to use; force transducer	front crawl maximal speed only
Planimetric method	≈ 1.5	Easy to use; force transducer (Dp) The (original) protocol is for research only	all strokes range of speeds (F vs. v)
Residual thrust method	≈ 2	The protocol is for research only and it is time consuming	front crawl range of speeds (F vs. v)
Biophysical method	≈ 2	The protocol is for research only and it is time consuming Metabolic data should also be collected	front crawl (breaststroke) range of aerobic speeds (F vs. v)
Sprint start method	≈ 2	The protocol is for research only and it is time consuming	Front crawl (Kass, Ma, Fpss)

Take home message

The propulsive force a swimmer can generate in water and the active drag can be assessed by means of the MRT method (based on full and semi-tethered tests)

The propulsive force a swimmer can generate in water and the active drag can be assessed by means of a standing start test (based on kinematic data and on values of added mass)

These two methods are in agreement: the propulsive force a swimmer can generate in water corresponds to the full tethered force and active drag is about twice than passive drag

Added mass is about 25% of body mass, both in passive and active conditions



ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA



Matteo Cortesi



Silvia Fantozzi



Vittorio Coloretti



ÉCOLE NATIONALE DES
PONTS
ET **CHAUSSÉES**



IP PARIS



Rémi Carmigniani