Beyond the Third Dimension

How Multidimensional Projections and Machine Learning Can Help Each Other



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A Bit of (Personal) History

before 1980

around 2000





Current Research



Software visualization



Simplified network visualization



High-dimensional data



Multiscale shape processing

Visual analytics for machine learning

What is really high-dimensional data?

Hundreds of dimensions with often no clear meaning

High-dimensional data Machine Learning 12:000 13:000 14:000 15:000 12:000 12:000 12:000 14:000 14:000 14:000 14:000 14:000 14:000 14:000 14:000 14:000 12:000 12:000 Հայուղ գետերին ը լորդությունն Անել օդրա արդ մարտարարությունն հետ կները, աններին արդ հետ հեներ **Solution: Multidimensional projections**

Projections



Why is this useful?

- no matter how large *n* is, we obtain a 2D scatterplot-like image (so it's visually scalable)
- point-to-point distance (in 2D) shows similarity of observations (in nD)
- coloring points by one attribute can show additional information on the observations

Machine Learning and Dimensionality Reduction Commonalities



Machine Learning and Dimensionality Reduction Commonalities



Machine Learning and Dimensionality Reduction Commonalities









DR for ML

Big Key Idea



- take two *n*D feature 2-class datasets A and B
- project them both to 2D using the same DR method
- you see the scatterplots above

Which dataset is easier to classify? Why?

Big Key Idea



You likely say B is easier. Why? Can we bring evidence for this?

Projections predict classification efficacy

Extensive set of experiments proved that separation in a (good) projection *predicts* accuracy of a (good) classifier



Bottom row: Easy classification (after feature selection)

Bottom row: Select 20 of 550 features on their discriminative power on training set, using extremely randomized trees P. Rauber et al. (2017) Projections as Visual Aids for Classification System Design. Inf Vis 17(4), 282-305

Projections: Central tool in classifier design



T1: Predict classification efficacyT2: Improve classification efficacy

Visual Active Learning

We have *under 5%* labeled samples How to train a classifier with such data?



parasites dataset colors = manually assigned labels black = unlabeled samples



Visual Active Learning: Bringing in the User

Hypotheses

- similar-label samples project close to each other
- decision boundaries are located in 'empty' area between clusters having different labels

Let's test this!

propag.



MAIST

Prot.c.

H.Eggs

H.Larvae(I)

H.Eggs(I)

Prot.c.(I)



Visual Active Learning: User and Machine Cooperate



- automatically label points where label-propagation confidence is high
- let user label (or not) low-confidence points

Propagation in 2D space is better than in nD!

Benato et al. (2020) Semi-automatic data annotation guided by feature space projection. Patt Recogn 109

Projections for understanding deep neural networks

Artificial Neural Networks (ANNs)





Challenges

- ANN is a true 'black box'
- when results are not optimal, how to
 - understand what has gone wrong, and where?
 - improve the classifier?

Explore learned representations (activations)

Method

- project input samples (images) having all activations in a layer as dimensions
- we see how training and the layer structure create information





Explore learned representations to improve classification



reasonable visual separation

• AC: 77.3%

- much better visual separation
 - AC: 93.8%

Explore learned representations to improve classification



reasonable visual separation

• AC: 77.3%

- much better visual separation
 - AC: 93.8%

Explore learned representations to improve classification

What is going on?

- visually explore clusters by brushing
- we find that each cluster-pair contains
 - a cluster for light images on dark background
 - a cluster for dark images on light background

Let's use this insight to improve the classification



Explore evolution of learned representations

Activation in an ANN change in time in two ways

- as data flows from the 1st to the last layer during operation (**inter-layer** evolution)
- as different datasets are used during training (inter-epoch evolution)

We want to explore both to

- understand how different layers contribute to learning
- understand if training is effective



Inter-layer evolution

Bundled observation paths (built using dynamic t-SNE)

We observe how

- group separation increases
- group size decreases
- groups increasingly diverge
- few trails connect different groups (classification decisions are stable)

Conclusions

Network performs (very) well!

MNIST dataset, MLP classifier

Explore evolution of learned representations

Inter-epoch evolution



Bundled observation paths

We observe how

- group separation increases (from complete clutter to perfect separation)
- groups increasingly diverge
- paths are quite straight/smooth (no canceling of learning)
- paths don't link different-color groups

Conclusions

- · learning is very effective
- knowledge accumulates as desired
- few/no 'learning hesitations'

MNIST dataset, last CNN hidden layer, 100 training epochs

Classifier Decision Maps

Often referred to in many ML papers, rarely shown



Question: How can we actually see those decision boundaries?

Classifier Decision Maps

Often referred to in many ML papers, rarely shown



Yes, we can visualize decision boundaries using projections!

Classifier Decision Maps



luminance: classifier confidence

luminance: distance to boundary

luminance: classifier confidence

Key to solution: Construct inverse projection from 2D to nD

Espadoto *et al.* (2021) UnProjection: Leveraging inverse-projections for visual analytics of high-dimensional data. IEEE TVCG Schulz *et al.* (2015) Using discriminative dimensionality reduction to visualize classifiers. Neural Process Lett 42(1)

Multi-classifier decision maps

Visualize agreement of nine classifiers on two-class problems



Classifiers used

handbags

- Logistic regression, Linear SVM (linear), SVM (RBF), kNN, Gaussian Process, Decision Tree, Random Forest, Adaboost, Gaussian Naive Bayes, Quadratic Discriminant Analysis
- t-SNE: small cluster separation but narrow white band (low uncertainty) ٠
- UMAP: large cluster separation but thick white band (high uncertainty) ٠

M. Espadoto et al. (2021) UnProjection: Leveraging Inverse-Projections for Visual Analytics of High-Dimensional Data. IEEE TVCG

Which projection technique is the "best"?

Projections are clearly useful tools for ML engineering But which projection is the *best* to use in practice?



Which DR technique to *use*?

Projection	Projection Full Name	Fodor	Hoffman	Yin	Maaten	Bunte	Engel	Sorzano	Cunningham	Gisbrecht	Liu	Xie	Nonato	Ours	
Acronym	Autoongodor	et al. [18]	et al. [1]	et al. [19]	et al. [13]	et al. [15]	et al. [27]	et al. [12]	et al. [23]	et al. [21]	et al. [2]	et al. [24]	et al. [10]	0.10	
CCA	CCA (Canonical Correlations Analysis)				•				•					•	
CHL	ClassiMap												•		
CuCA	CCA (Curvilinear Component Analysis)												•		techniques
DML	Distance Metric Learning				•				•					•	teonniques
EM FA	Elastic Maps Factor Analysis	•						•	•					•	
FD	Force-Directed												•		
FNIAF	Feature Selection											•	•	•	
GDA GPLVM	Generalized Discriminant Analysis Gaussian Process Latent Variable Model													•	_
GTM	Generative Topographic Mapping							•	•				•		
F-ICA	FastICA	-						-	-					•	Big and unclear 'choice space'
IDMAP	IDMAP	•												•	
ISO L-ISO	Isomap Landmark Isomap		•	•	•	•	•			•			•	•	
KECA	Kernel Entropy Component Analysis							•							
LAMP	LAMP												•	•	• 50+ techniques
LDA	Linear Discriminant Analysis Laplacian Eigenmaps				•	•			•	•	•	•	•	•	
LLC	Locally Linear Coordination Locally Linear Embedding		•	•	•	•	•			•	•		•	•	 12 main surveys
H-LLE M-LLE	Hessian LLE Modified LLE				•									•	
LMNN	Large-Margin Nearest Neighbor Metric													•	 mainly theoretical discussion
LOCH	Local Convex Hull Locality Preserving Projection								•				•	•	
LR LSP	Linear Regression Least Square Projection								•				•	•	 many parameters
LTSA	Local Tangent Space Alignment				•								•	•	
MAF	Maximum Autocorrelation Factors								•						 very limited practical comparison
MCA	Manifold Charting Multiple Correspondence Analysis				•					•			•	•	
MCML MDS	Maximally Collapsing Metric Learning Metric Multidimensional Scaling	•	•	•	•	•	•	•	•		•		•	•	_
L-MDS MG_MDS	Landmark MDS Multi-Grid MDS													•	
N-MDS	Nonmetric MDS (Kruskal)		•				•						•	•	Practitioner questions
MVU	Maximum Variance Unfolding				•	•	•			•			•		
FMVU L-MVU	Fast MVU Landmark MVU													•	_
NeRV t-NeRV	Neighborhood Retrieval Visualizer t-NeRV					:									
NMF	Nonnegative Matrix Factorization							•	•				-	•	• which projection is best for my
NN	Neural Networks	•											•		
PBC PC	Projection By Clustering Principal Curves	•						•						•	contaxt (requirements data)?
PCA I-PCA	Principal Component Analysis Incremental PCA	•	•		•		•	•	•	•	•	•	•	•	
K-PCA-P	Kernel PCA (Polynomial)									-				•	• how to sot its paramotors ?
K-PCA-S	Kernel PCA (Kbr) Kernel PCA (Sigmoid)		-		•		•	-		•				÷	
NL-PCA	Localized PCA Nonlinear PCA	•		•				•							• how to moscure its quality?
P-PCA R-PCA	Probabilistic PCA Robust PCA							•	•					•	• now to measure its quality?
S-PCA PLMP	Sparse PCA Part-Linear Multidimensional Projection		-					•					-	•	
PLP	Piecewise Laplacian-based Projection						•								
PLSP	Principal Manifolds			•										•	
PP RBF-MP	Projection Pursuit RBF Multidimensional Projection	•											•		
RP C-RP	Random Projections	•										•			
S-RP	Sparse Random Projection												-	•	
R-SAM	Rapid Sammon (Pekalska)				•								•	•	
SDR SFA	Sufficient Dimensionality Reduction Slow Feature Analysis								•						
SMA	Smacof Stochastic Neighborhood Embedding												•		
T-SNE	t-Dist. Stochastic Neighborhood Embedding					•				•	•			•	
ViSOM	ViSOM (Visualization-induced SOM)	•		•				•					•		
SPE G-SVD	Stochastic Proximity Embedding Generalized SVD							•						•	
T-SVD TF	Truncated SVD Tensor Factorization							•						•	
UMAP	Uniform Manifold Approximation and Proj.	.	-										-	•	
Total	recor Quantization	12	6	7	14	9	9	19	14	8	6	4	28	44	

surveys

Let's measure projection errors big-scale!



M. Espadoto *et al* (2019) Towards a Quantitative Survey of Dimension Reduction Techniques (IEEE TVCG)

Datasets and Metrics

Datasets

Dataset	Type	Size	Size	Dimensionality	Dimensionality	Intrinsic	Intrinsic	Sparsity	Sparsity
	(au_D)	(N)	class	(n)	class	dim. (ρ_n)	dim. class	(γ_n)	class
bank	tables	2059	medium	63	low	0.0317	low	0.6963	medium
cifar10	images	3250	large	1024	high	0.0706	low	0.0024	dense
cnae9	text	1080	medium	856	high	0.3201	medium	0.9922	sparse
coil20	images	1440	medium	400	medium	0.0105	low	0.3858	medium
epileptic	tables	5750	large	178	medium	0.2191	medium	0.0067	dense
fashion_mnist	images	3000	medium	784	high	0.2385	medium	0.5021	medium
fmd	images	997	small	1536	high	0.3073	medium	0.0095	dense
har	tables	735	small	561	high	0.1194	medium	0.0001	dense
hatespeech	text	3222	large	100	medium	0.6130	high	0.9993	sparse
hiva	tables	3076	large	1617	high	0.2498	medium	0.9091	sparse
imdb	text	3250	large	700	high	0.5790	high	0.9945	sparse
orl	images	400	small	396	medium	0.0006	low	0.9000	sparse
secom	tables	1567	medium	590	high	0.0102	low	0.2617	medium
seismic	tables	646	small	24	low	0.0417	low	0.5883	medium
sentiment	text	2748	medium	200	medium	0.8080	high	0.9936	sparse
sms	text	836	small	500	medium	0.7240	high	0.9947	sparse
spambase	text	4601	large	57	low	0.0351	low	0.7741	medium
svhn	images	733	small	1024	high	0.8734	high	0.0001	dense

Metrics

Metric	Definition	Туре	Range
Trustworthiness (M_t)	$1 - \frac{2}{NK(2n-3K-1)} \sum_{i=1}^{N} \sum_{j \in U_i^{(K)}} (r(i,j) - K)$	scalar	[0, 1]
Continuity (M_c)	$1 - \frac{2}{NK(2n-3K-1)} \sum_{i=1}^{N} \sum_{j \in V_i}^{i} (\hat{r}(i,j) - K)$	scalar	[0, 1]
Normalized stress (M_{σ})	$\frac{\sum_{ij} (\Delta^n(\mathbf{x}_i,\mathbf{x}_j) - \Delta^q(P(\mathbf{x}_i),P(\mathbf{x}_j)))^2}{\sum_{ij} \Delta^n(\mathbf{x}_i,\mathbf{x}_j)^2}$	scalar	[0 ,1]
Neighborhood hit (M_{NH})	$\sum_{i=1}^{N} rac{ j \in N_i^{(K)}: l_j = l_i }{KN}$	scalar	[0, 1]
Shepard diagram (S)	Scatterplot $(\mathbf{x}_i - \mathbf{x}_j , P(\mathbf{x}_i) - P(\mathbf{x}_j)), 1 \le i \le N, i \ne j$	point-pair	-
Shepard goodness (M_S)	Spearman rank correlation of Shepard diagram	scalar	[0, 1]
Average local error $(M_a(i))$	$\frac{1}{N-1}\sum_{j\neq i}\left \frac{\Delta^{n}(\mathbf{x}_{i},\mathbf{x}_{j})}{\max_{i,j}\Delta^{n}(\mathbf{x}_{i},\mathbf{x}_{j})}-\frac{\Delta^{q}(P(\mathbf{x}_{i}),\tilde{P}(\mathbf{x}_{j}))}{\max_{i,j}\Delta^{q}(P(\mathbf{x}_{i}),P(\mathbf{x}_{j}))}\right $	local (per-point)	[0 ,1]

aggregate into a single quality metric μ



Insights (1)

How good are projections, for which data?

for each projection P_i for each dataset D_j compute *optimal* quality μ_{ij} (param. grid search)

How easy is to get optimal quality?

for each projection P_i compute *variance* of params π_i yielding optimal quality over all datasets D_j

What we see

- no projection best for all dataset types
- some are quite **poor** in general (N-MDS, GDA)
- dataset type strongly influences quality (*imdb*: hard; *orl*: easy)
- hard to **tune** parameters to get optimal quality (large variance of π_i)



Insights (2)

How good are parameter-preset projections?

for each projection P_i π_i^{pre} = param values yielding most times optimal quality over all datasets D_j

for each projection P_i for each dataset D_j compute quality μ_{ij} using $\pi_i{}^{pre}$

What we see

- very similar image to earlier one (optimal techniques stay good when using presets)
- again, quality strongly depends on dataset type
- t-SNE, UMAP, IDMAP, PBC score best on average

Insights (3): Which projections perform similarly?



'Projection of projections' map

- one point = one technique
- 5 attributes (trustworthiness, continuity, norm. stress, neighborhood hit, Shepard goodness; averaged over all tested datasets)
- we see a clear quality trend
- helps choosing projections that behave similarly to a user-chosen one

Benchmark

Towards A Quantitative Survey of Dimension Reduction Techniques

MATEUS ESPADOTO, RAFAEL M. MARTINS, ANDREAS KERREN, NINA S. T. HIRATA AND ALEXANDRU C. TELEA

DATASETS EXPERIMENT MEASUREMENTS PROJECTIONS

Projections for all datasets (best parameter set for each projection)

All projections, in csv format



All open source

- projection implementations
- datasets
- metric engines
- visualization engines
- optimization engines
- test harness
- all Python code

Please share, use, and extend!

https://mespadoto.github.io/proj-quant-eval



ML for DR

Insights from our DR survey

No ideal projection technique 🛞

- UMAP: easy to use, quite fast, but quality not ideal
- t-SNE: quality is (very) high, but very slow, hard to tweak parameters, non-deterministic
- quality depends a lot on type of data

What we want to ultimately have

- high-quality projection
- having `style' of any projection deemed good by user
- working very fast (millions of samples, hundreds of dimensions: seconds)
- **easy** to use (no complex parameters, ideally none)
- **stable** (same input data: same output projection)
- **out-of-sample** (add some more data: project along existing data)

How to achieve this?

Idea: Learn the projection!



- take any dataset D_S and any projection technique of choice P
- project D_S with P, tweak P's parameters, obtain good scatterplot P(D_S)
- pass D_S and P(D_S) to network, learn the mapping
- use trained network P_{nn} to project any other similar dataset D_P

Espadoto et al. (2020) Deep learning multidimensional projections. Inf Vis 9(3):247-269

Learning different projection styles



- we can imitate basically any style
- but, of course, the output quality will depend on the training material's quality (good `professor' = good quality, and conversely ⁽ⁱⁱⁱ⁾)

Learning different projection styles (cont'd)



Learning different projection styles (cont'd)



Learning different projection styles (cont'd)





Out of sample capability

Testing

- train on a dataset D₀
- add samples to D₀ to create D₁, D₂, ...D_n
- project $P(D_0),...,P(D_n)$
- compare with ground-truth P^g(D₀),... P^g(D_n)

Results

- our method is always stable (out-of-sample capability by construction)
- most other methods are **not**
- we are close to the quality of parametric t-SNE (pt-SNE)

Computational scalability



Training + inference costs

• **3K faster** than t-SNE, 2K faster than LAMP

UMAP, LSP, MDS failed handling 1M points

Inference-only costs

- 3.5K faster than t-SNE, 2K faster than LAMP
- 10x faster than pt-SNE

Code freely available: https://github.com/mespadoto/dlmp

kNNP: Improve NNP by training on *neighborhoods*



T. Modrakowski et al. (2021) Improving Deep Learning Projections by Neighborhood Analysis. Springer CCIS 50

Intermezzo: Sharpening Data by Mean Shift (MS)



Mean shift is a very powerful, generic, tool for finding clusters in a data distribution!

D. Comaniciu, P. Meer (2002) Mean shift: a robust approach toward feature space analysis. IEEE TPAMI 24(5), 603-619

Mean Shift for Projections



Mean Shift for Projections



- for DR methods which cannot easily separate clusters well, sharpening brings a lot (LMDS, RP)
- for methods which are already sharp, we don't gain much (t-SNE, UMAP)
- this is exactly the idea: make simpler/faster DR methods deliver better!



The Way Forward

ML for DR: How to measure the quality of inverse projections?

Gradient maps: Show areas where not to trust the (inverse) projection



warm colors: tears in projection

A general framework for inverse-projection errors is missing!

Espadoto et al. (2021) UnProjection: Leveraging inverse-projections for visual analytics of high-dimensional data. IEEE TVCG

ML for DR: How to project time-dependent data?

	s	k	Dista preser	ance vation	I	N F	eighb oreser	orhoo vation	d I		Temporal stability				
ľ	Methods	Spearson	Sspearman	SKendall	Sstress	S _{NH}	S_{NP}	STrust	S _{Cont}		T _{Pearson}	T _{Spe} arman	$T_{Kendall}$	T _{Stress}	
coders	AE	0.740	0.804	0.659	0.519	0.588	0.497	0.907	0.879		0.486	0.672	0.564	1.026	
Autoen	VAE	0.760	0.803	0.659	0.479	0.583	0.493	0.895	0.875		0.549	0.685	0.581	0.900	
spou	TF-t-SNE	0.477	0.577	0.442	1.045	0.592	0.573	0.921	0.902		0.020	0.002	0.002	1.959	
od met	G-t-SNE	0.660	0.704	0.531	0.679	0.549	0.432	0.816	0.808		0.329	0.487	0.386	1.340	
borhoo	dt-SNE	0.609	0.675	0.514	0.781	0.479	0.408	0.808	0.797		0.184	0.192	0.147	1.631	
h/neigh	TF-UMAP	0.497	0.617	0.472	1.005	0.572	0.542	0.907	0.884		0.119	0.089	0.060	1.760	
Grap	G-UMAP	0.610	0.672	0.502	0.779	0.508	0.387	0.771	0.779		0.264	0.316	0.234	1.471	
riants	TF-PCA	0.784	0.810	0.669	0.431	0.544	0.493	0.917	0.874		0.312	0.453	0.354	1.374	
PCA va	G-PCA	0.778	0.805	0.665	0.442	0.546	0.485	0.904	0.867		0.586	0.673	0.580	0.827	
low quality															

Only a handful of dynamic projection techniques exist Visual quality and stability seem to be mutually exclusive goals (!)

ML for DR: How to project time-dependent data?



Visual quality and stability seem to be mutually exclusive goals (!)

DR for ML: Visualizing general regressors



visualizing optimizers

How to visualyze any nD-to-mD regressor?

Espadoto et al. (2021) OptMap: Using Dense Maps for Visualizing Multidimensional Optimization Problems. Proc. IVAPP Espadoto et al. (2023) Visualizing High-Dimensional Functions with Dense Maps. Springer CCIS 68

DR for ML: More information in a projection



explain projection structures by dimension values

encode information in projection shape

A projection should tell a rich, complete, insightful story about the data!

Thank you for your interest!

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- examples, applications
- code
- datasets
- papers
- people and projects

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