A Heuristic Algorithm for the Inventory Routing Problem with Logistic Ratio

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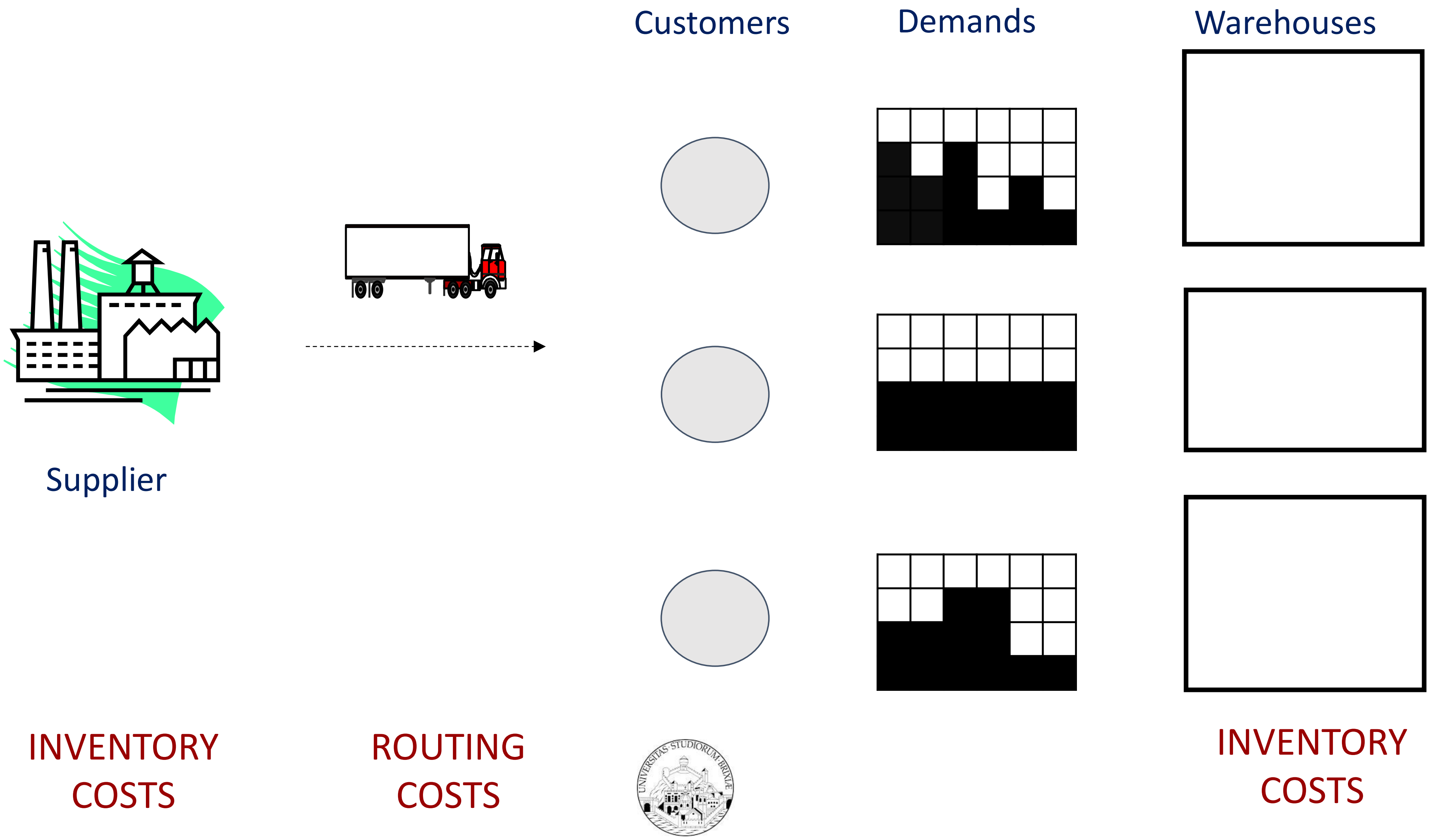
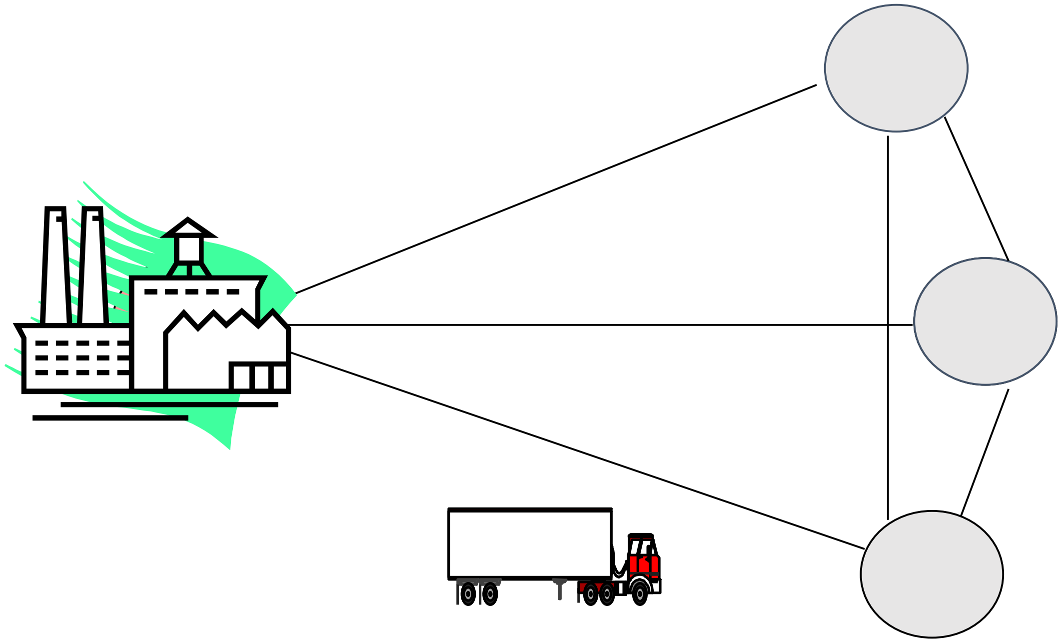
Outline

- Introduction to the IRP
- Why?
- Logistic ratio
- ILS
- Results
- Conclusions

Inventory Routing Problem (IRP)



Inventory Routing Problem (IRP)



Find

a distribution plan over a **planning horizon**

that minimizes

routing costs

and inventory holding costs

VMI vs. RMI approaches

- VMI: the supplier has full control of decisions
  - GOAL: minimize the total system cost
- RMI: each retailer decides when to be replenishes and how much
  - GOAL: minimize retailer's inventory cost
- Archetti and Speranza (ITOR 2016) showed that VMI can provide savings of up to 20%

IRP formal definition

- **Directed** complete graph G=(N,A), where N= 0 (supplier, depot)∪ N′ (customers)
- n customers
- T set of time periods {1,…,H}, H horizon
- Fleet K of m homogeneous vehicles with capacity Q
- Maximum inventory level at customers U_i
- **Split deliveries are not allowed**
- Routing cost c_{ij} that satisfy triangle inequality
- Inventory cost at customers and supplier h_i

IRP formal definition

Find the distribution plan:

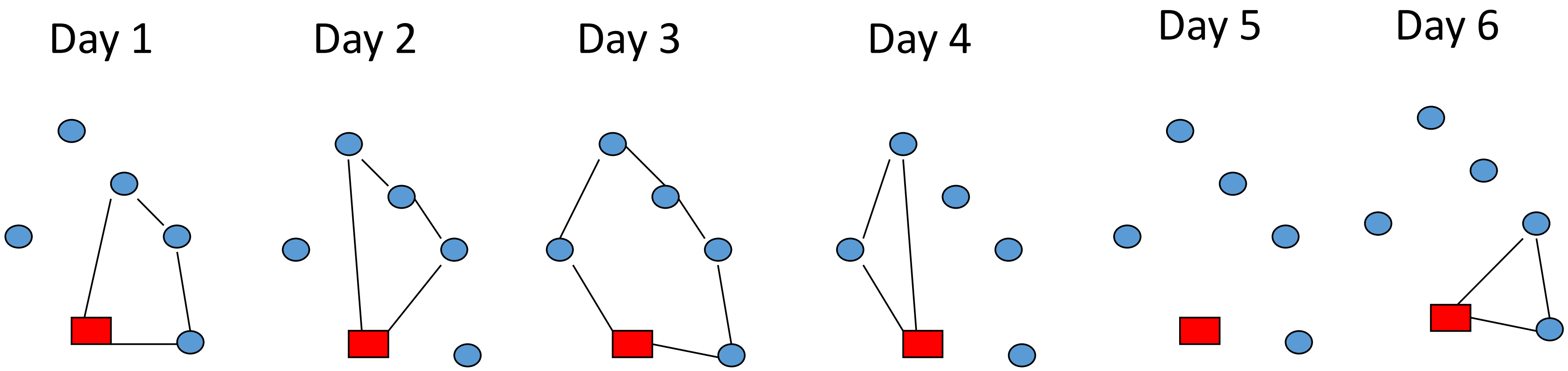
Delivery schedule

+

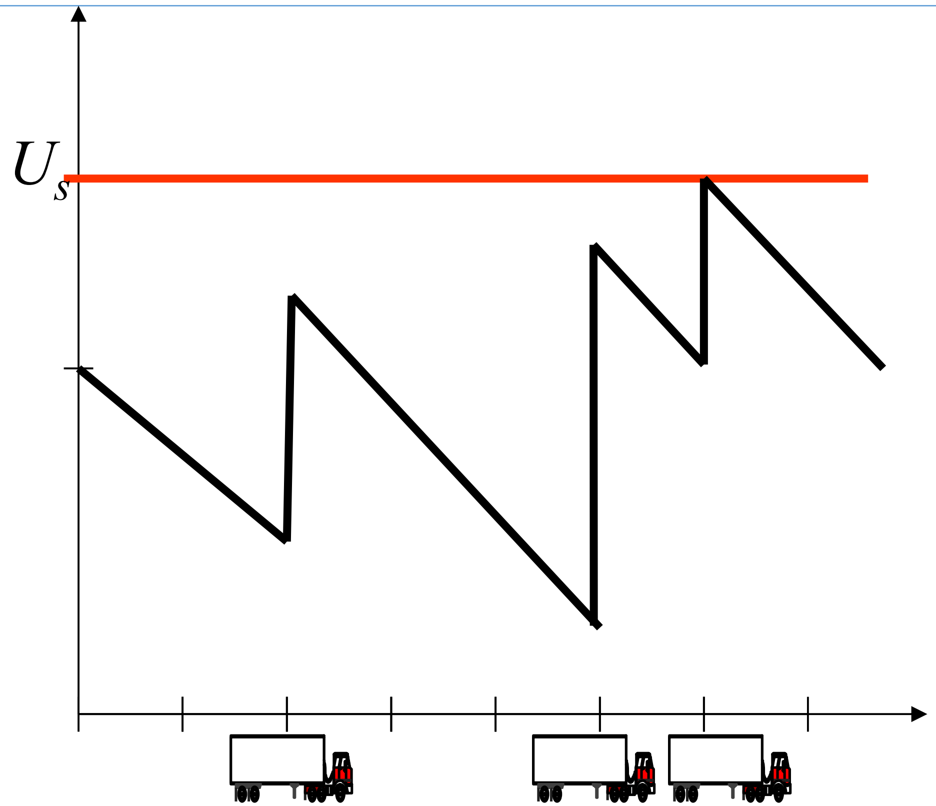
Routing

Minimizing the total cost: routing + inventory

Example



Example



HOWEVER…

Practical issue

- What does the inventory holding cost represent?

- How can you determine its value?

… Its is often hidden or difficult to estimate….

Also…

- When the inventory cost is negligible w.r.t. transportation cost, ending inventory level is mostly null
- But this has a huge impact on the next planning period!

So?

- Add constraints on ending inventory level

OR

- New objective function

Logistic ratio

- It corresponds to ratio the between the total transportation (routing cost) and the total quantity
- Often used in practice (ROADEF Challenge 2016: https://roadef.org/challenge/2016/en/)
- Applications in chemical industry (oil distribution), mining industry…

Mathematical formulaiton

- Objective function:
  - Minimize LR: Routing cost / Total quantity delivered to customers
- Constraints (classical IRP constraints):
  - Inventory constraints
  - Vehicle capacity constraints
  - Routing constraints

Mathematical formulation

Variables

- I_i^t: continuous inventory variables
- q_{it}^k: continuous quantity variables
- z_{it}^k: binary visiting variables associated with customers and depot
- y_{ij}^t: binary routing variables

Mathematical formulation

$$\min \quad \sum_{k\in K} \sum_{<i,j>\in E} \sum_{t\in T} c_{ij} y_{ij}^{kt} \Big/ \sum_{k\in K} \sum_{i\in N'} \sum_{t\in T} q_{it}^{k}$$

Objective function

$$\text{s.t.} \quad I_{0t} = I_{0,t-1} + r_{0t} - \sum_{k\in K} \sum_{i\in N'} q_{it}^{k} \qquad t \in T$$

$$I_{it} = I_{i,t-1} - r_{it} + \sum_{k\in K} q_{it}^{k} \qquad i \in N',\ t \in T$$

Inventory constraints

$$I_{it} \geq 0 \qquad i \in N,\ t \in T$$

$$\sum_{k\in K} q_{it}^{k} \leq U_i - I_{it-1} \qquad i \in N',\ t \in T$$

$$q_{it}^{k} \leq U_i z_{it}^{k} \qquad i \in N',\ k \in K,\ t \in T$$

$$\sum_{i\in N'} q_{it}^{k} \leq Q z_{0t}^{k} \qquad k \in K,\ t \in T$$

$$\sum_{k\in K} z_{it}^{k} \leq 1 \qquad i \in N',\ t \in T$$

Capacity and no-split constraints

Mathematical formulation

$$\sum_{j:<i,j>\in E} y_{ij}^{kt} = 2z_{it}^{k} \qquad i \in N,\ k \in K,\ t \in T$$

Routing constraints

$$\sum_{<i,j>\in E(S)} y_{ij}^{kt} \le \sum_{i\in S} z_{it}^{k} - z_{st}^{k} \quad S \subseteq N',\ s \in S,\ k \in K,\ t \in T$$

$$z_{it}^{k} \in \{0,1\} \qquad i \in N,\ k \in K,\ t \in T$$

$$q_{it}^{k} \ge 0 \qquad i \in N',\ k \in K,\ t \in T$$

$$y_{ij}^{kt} \in \{0,1\} \qquad <i,j>\in E,\ k \in K,\ t \in T$$

$$y_{0j}^{kt} \in \{0,1,2\} \qquad j \in N',\ k \in K,\ t \in T.$$

Variables domain

Literature review

- **ROADEF Challenge 2016:** Special Section in TS in 2020
- **Exact Approaches:** Archetti, Desaulniers, Speranza (EJTL 2017), Archetti, Coelho, Speranza (TRE, 2019)
- **Heuristic Approaches:** Alvarez, Munari, Morabito (ITOR 2018)

Heuristic algorithm: ILS

Why ILS?

- **SIMPLE** and extremely **effective** on many combinatorial optimization problems and especially in routing (Lorenço, Martin, Stuezle, Handobook of Metaheuristics, 2018)

NOTE: Alvarez et al (2018) proposed an ILS as well (and also SA)

ILS: General Scheme

procedure *Iterated Local Search*
    s_0 = GenerateInitialSolution
    s^* = LocalSearch(s_0)   % optional
    repeat
        s' = Perturbation(s^*)
        $s^{*\prime}$ = LocalSearch(s')
        s^* = AcceptanceCriterion($s^*, s^{*\prime}$)
    until termination condition met
end

ILS-LR

Algorithm 2 ILS-LR

```
s* ← Empty and f(s*) ← +∞
while maxIter do
    s ← Construct initial schedule
    s ← HGS(s) {Improve initial routing}
    s* ← s
    while ≤ maxLS do
        s ← Local_Search(s)
        if s is feasible then
            s ← Rebalance(s)
            if f(s) < f(s*) then
                s* ← s
            end if
        end if
        if criterion met then
            Update Penalty
        end if
    end while
    s* ← HGS(s*)
end while
return solution s*
```

ILS-LR: solution space (Update Penalty)

- Infeasibility is allowed in terms of **STOCK-OUT ONLY**

- Adaptive penalty term

ILS-LR: Construct initial schedule

- Customers are randomly ordered
- Each customer is served when stock-out is faced, if feasible (capacity), otherwise before
- Maximum feasible quantity (max. level, capacity) is delivered each time

ILS-LR: Improving routing cost

- HGS from Vidal et al (2012):

    1. State-of-the-art heuristic for many routing problems
    2. Flexible and easy to use
    3. Open-source code

- No change to the visiting and replenishing schedule

- Applied on each day of the planning horizon (VRP)

ILS-LR: Local search

- Classical LS operators from routing
  - Insertion
  - Removal
  - Swap
- Max quantity is delivered at each insertion

ILS-LR: Rebalance operator

- It is applied only on feasible solutions
- Aims at improving solution values by modifying deliveries quantities across all customers (contrary to single-customer operations as in LS)
- Tailored to LR: the higher the quantity (with constant routing cost), the better the LR

ILS-LR: Rebalance operator

Sketch:

- Refresh all delivery quantities to minimum value to avoid stock-out (visiting schedule and routing are not modified)
- For each day:
  - Order customers by PRIORITY RATE (low inventory value, high demand, high interval to next visit)
  - Increase delivered quantity as much as possible to customers according to the order

ILS-LR vs ILS from Alvarez et al.

- Stock-out infeasibility is allowed
- HGS for routing
- Rebalance operator
- Alvarez et al. use many more LS operators

Computational tests

Benchmark IRP **SMALL** instances:

- *H* = 3, 6
- *n* = 5, 10, …., 30 (H=6) … 50 (H=3)
- *m* = 1 – 5
- 400 instances

Comparison with Alvares et al. (best approach between ILS and SA) - AMM

Results: Improvement over AMM – Horizon 3

	m=1	m=2	m=3	m=4	m=5	Average
n=5	-13,86	-25,96	-27,56	-24,33	-21,92	-22,72
n=10	-16,09	-12,90	-18,81	-18,62	-17,38	-16,76
n=15	-17,31	-9,28	-15,04	-16,56	-14,88	-14,62
n=20	-16,56	-13,48	-12,57	-15,21	-14,10	-14,38
n=25	-7,53	-9,75	-10,04	-11,16	-10,25	-9,75
n=30	-16,15	-17,37	-12,78	-12,77	-8,86	-13,58
n=35	-13,38	-10,67	-9,19	-12,45	-8,91	-10,92
n=40	-1,57	-8,54	-9,67	-9,68	-6,14	-7,12
n=45	-15,88	-9,95	-11,55	-9,00	-7,37	-10,75
n=50	-2,40	-9,02	-4,41	-3,49	-4,18	-4,70
Average	**-12,07**	**-12,69**	**-13,16**	**-13,33**	**-11,40**	**-12,52**

Results: Improvement over AMM – Horizon 6

	m=1	m=2	m=3	m=4	m=5	Average
n=5	-10,48	-12,22	-14,70	-15,32	-12,57	-13,08
n=10	-3,79	-5,26	-8,11	-9,05	-9,00	-7,04
n=15	5,07	-4,09	-1,04	-7,11	-5,66	-2,57
n=20	8,75	1,10	-1,10	-0,24	-3,51	1,00
n=25	5,36	3,03	1,88	4,17	2,61	3,41
n=30	10,66	1,81	1,46	2,62	-2,81	2,75
Average	**2,59**	**-2,61**	**-3,60**	**-4,15**	**-4,90**	**-2,56**

Results: Improvement over AMM

- Better solution on all 318 instances over 399 (1 instance is infeasible)

- MAX/AVER./MIN Improvement: **20.13/-0.07/-41.76**

Results: Computational time

	m=1		m=2		m=3		m=4		m=5		Average	
	AMM	ILS	AMM	ILS	AMM	ILS	AMM	ILS	AMM	ILS	AMM	ILS
n=5	11,54	8,30	21,56	11,20	21,62	11,60	26,18	12,90	28,80	14,78	**21,80**	**11,69**
n=10	17,56	10,60	22,83	14,90	24,22	16,80	27,21	19,40	28,87	18,60	**24,14**	**16,06**
n=15	22,25	12,80	23,60	16,90	24,70	18,40	28,32	23,30	29,93	24,40	**25,76**	**19,16**
n=20	24,18	13,70	25,44	20,90	26,08	24,80	29,70	26,90	30,01	29,70	**27,08**	**23,20**
n=25	25,25	14,80	26,78	24,50	27,44	27,80	30,02	34,60	30,03	35,10	**27,91**	**27,36**
n=30	27,54	20,00	28,67	30,50	29,59	36,00	30,03	42,80	30,05	41,40	**29,17**	**34,14**
n=35	29,14	13,40	30,01	21,00	30,01	23,20	30,01	28,20	30,01	31,20	**29,83**	**23,40**
n=40	30,00	18,20	30,00	25,80	30,00	25,60	30,01	29,00	30,01	43,60	**30,01**	**28,44**
n=45	30,01	18,00	30,01	27,80	30,01	33,60	30,01	31,60	30,01	42,00	**30,01**	**30,60**
n=50	30,01	20,20	30,01	31,60	30,01	37,20	30,01	53,60	30,02	65,00	**30,01**	**41,52**
Average	**23,49**	**14,39**	**26,11**	**21,50**	**26,71**	**24,40**	**28,93**	**28,89**	**29,73**	**32,08**	**26,99**	**24,23**

AMM is capped at 30 seconds

Conclusions

- An extremely challenging problem combining DISTRIBUTION and INVENTORY MANAGEMENT

- The difficulty arises from:
  - Time dimension
  - Combination of integer and continuous decision variables

Conclusions

- This is valid in general for the IRP

- IRP-LR adds the complexity of a fractional objective function

HOWEVER

- Extremely relevant in practice

Conclusions

- Only one benchmark heuristic from the literature
- ILS-LR is competitive on small instances but struggles when the dimension increases (on benchmark large instances it is not competitive)

Future directions

- Improve ILS-LR by introducing
    - Diversification mechanism
    - Tailored operator like the one proposed in Archetti et al. (2025 – EJOR) for the IRP

THANK YOU FOR YOUR ATTENTION