Rationality and the Bayesian Paradigm

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Does Rationality imply Bayesianism?

- Most economic theorists: yes
- Will try to challenge that
- We'll need to define the concepts
  - With minor digressions
- Implications for economic theory

Rationality

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?
- Rationality as robustness
- Weaknesses (?): subjective, empirical, not monotonic in intelligence
- Defense

Digression I: Objectivity and Subjectivity

- Anscombe-Aumann
- Schmeidler's example
- Objectivity as second-order subjectivity
- Habermas's notion of "communicative rationality"

Objective and Subjective Rationality

- A decision maker is defined by two relations $(\succsim^*, \succsim^\wedge)$
- $\succsim^*$ – can convince "any reasonable decision maker" that it is right
- $\succsim^\wedge$ – cannot be convinced that it is wrong
- Naturally, $\succsim^* \subset \succsim^\wedge$

The Bayesian Approach

- Formulate state space
- All uncertainty resolved by the state
- Formulate a prior probability
- Update by Bayes’s rule

Subjective Probabilities

Pascal, one of the founders of probability theory (if not *the* founder)

for games of chance (“risk”)

suggested subjective probabilities and expected utility maximization in his “Wager”

Blaise Pascal (1623-1662)

Pascal's "Wager"

	God is	God is not
Become a believer	∞	0
Forget about it		0

The basic argument: what have you got to lose?

What we call today "a (weakly) dominant strategy"

What Pascal didn't say

	God is	God is not
Become a believer	∞	0
Forget about it	$-\infty$	0

While some use burning in hell to scare you into faith, Pascal believed in positive marketing

Beyond dominance

	God is	God is not
Become a believer	∞	0
Forget about it		c

… But even if there is some $c > 0$ that you have to give up on by becoming a believer, it's finite.

Hence it's better to become a believer

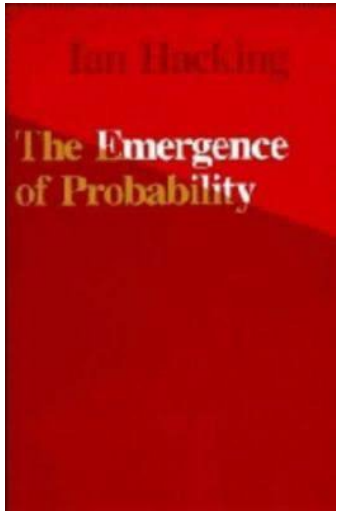
Pascal's Wager – Ideas

A few ideas in decision theory (Hacking, 1975)

- The decision matrix
- Dominance
- Subjective probability
- Expected utility
- Absence of probability

Not to mention humanism… (Connor, 2006)

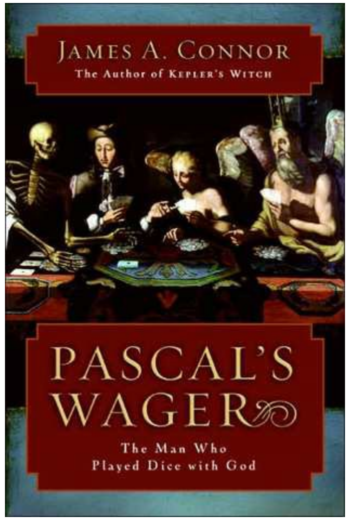
The emergence of probability



Ian Hacking (b. 1936)

Pascal's Wager:

The man who played dice with God



James A. O'Connor

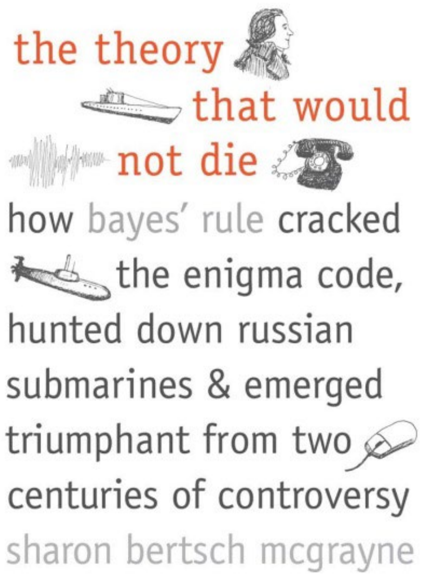
And then came Bayes

Bayesian updating is called after…

Rev. Thomas Bayes (1702-1761)

"Bayesian" – committed to having a subjective prior probability over any unknown

Relying on



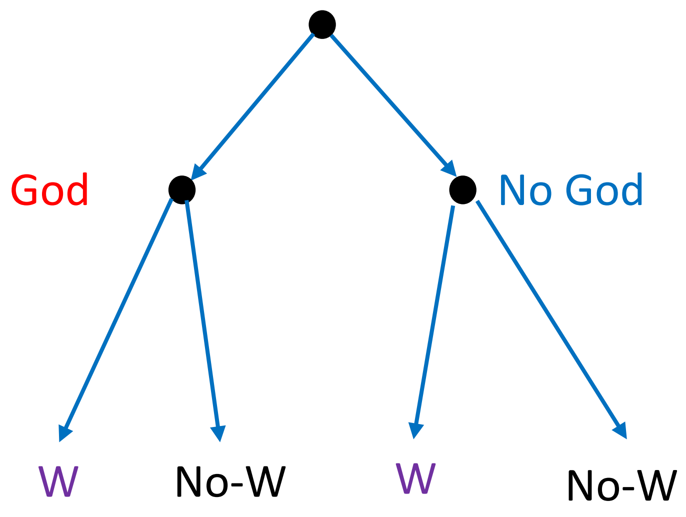
Sharon B. McGrayne (b. 1942)

The existence of God

As described in McGrayne (2011),

Bayes wanted to prove it

W – the World as we know it.



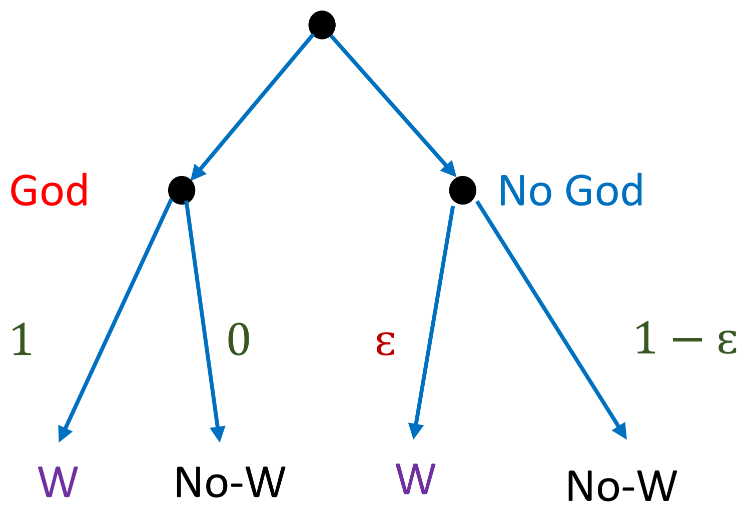
The conditional probabilities

If God exists, we'll surely find the World as we know it.

If not, lots of coincidences need to be assumed

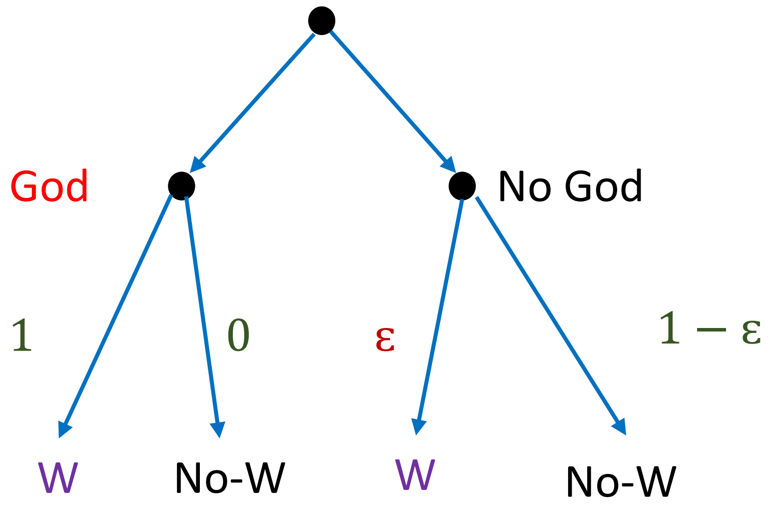
Their probability is, say, a small

$$\varepsilon > 0$$

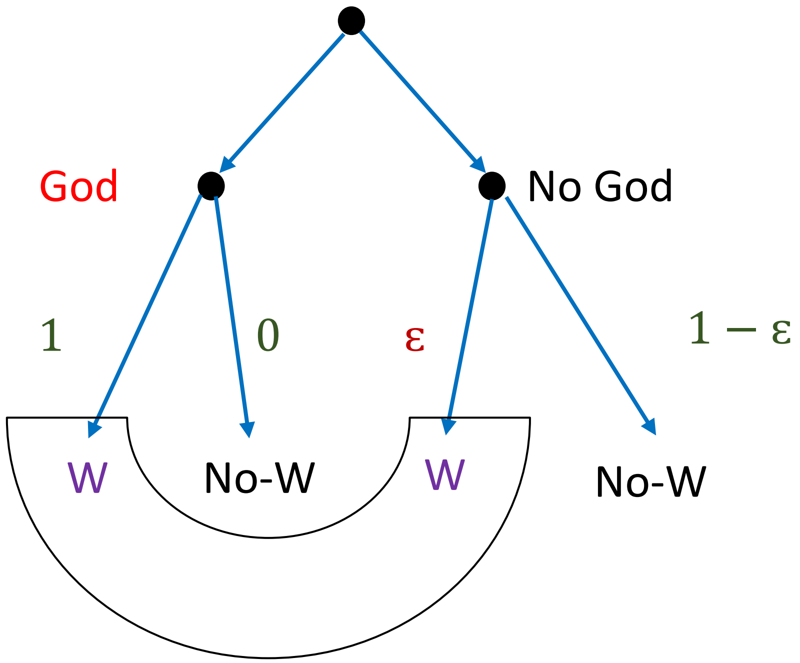


But we want the other direction…

$$P(\text{God}|\text{World}) \neq P(\text{World}|\text{God})$$



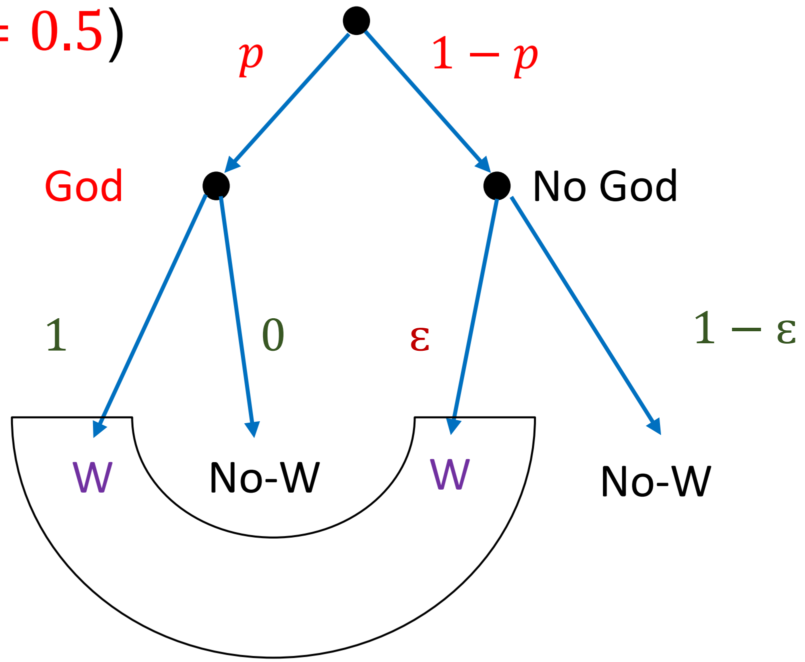
We need to collect probabilities



And we need a prior!

And then the argument can be completed

(Bayes used $p = 0.5$)



Digression II: Classical Statistics

- Say, hypothesis tests:
  - H_0: the defendant is innocent
  - H_1: the defendant is guilty
- No probability on either hypothesis
  - “significance”, “confidence” – derived from but are not probabilities
- The Bayesian alternative

Reconciling Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches
- Classical – for making a point (to others)
- Bayesian – for making a decision (for oneself)

The question, then…

- As mentioned above, Pascal (1670) already used (subjective) probabilities to discuss the problem of becoming a believer
- Bayes (1763) used subjective probabilities to convince us that God exists
- Classical statisticians refuse to treat parameters as random variables
- Bayesian statisticians insist that we do so

The question is:

Can we always quantify uncertainty probabilistically?

Answer 1: No, we cannot

Knight distinguished between these two types of uncertainty

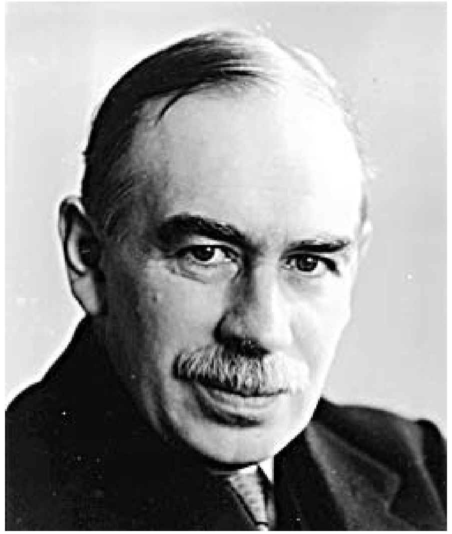
"Knightian uncertainty" – cannot be quantified

Argued that entrepreneurs are more tolerant of that type uncertainty

Frank Knight (1885-1972)

And this was also Keynes's view

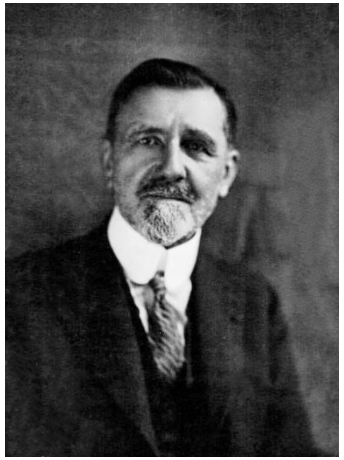
"...About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know."



John Maynard Keynes (1883-1946)

On the other hand…

There were others



Émile Borel
(1871-1956)

Frank P. Ramsey
(1903-1930)

Bruno de Finetti
(1906-1985)

Subjective probabilities and choice

- Borel, Ramsey, de Finetti: subjective probability should be measured by betting behavior

  "Put your money where your mouth is"

- Coherent betting behavior → maximization of expectation
- Define probabilities by behavior

Axiomatizations

- Conditions on presumably observed data that imply certain models
- Observability – along the lines of logical positivism (see Moscati)
- For example:
  - $\succsim$ complete and transitive $\Leftrightarrow$ can be represented by $\max u$

    (up to details)
- These are rhetorical results
  - Like existence, impossibility
  - Part of the discourse of theorists

Leonard J(immie) Savage

- An amazing theorem that shows

Leonard J Savage (1917-1971)

- Coherent decisions → expected utility maximization
- Both the probability and the utility are derived from preferences
- It convinced the entire field. Make it "fields".

The Bible (Savage, 1954)

$$F = X^S = \{f \mid f : S \to X\}$$

- P1 $\succsim$ is a weak order
- P2 $f_{A^c}^{h} \succsim f_{A^c}^{h} \quad iff \quad f_{A^c}^{h'} \succsim g_{A^c}^{h'}$
- P3 $x \succsim y \; iff \; f_A^x \succsim f_A^y$ whenever A is not null
- P4 $y_A^x \succsim y_B^x \; iff \; w_A^z \succsim w_B^z$ whenever $x \succ y, z \succ w$
- P5 $\exists \, f \succ g$
- P6 $f \succ g \Rightarrow \exists$ partition of $S, \{A_1, \ldots, A_n\}, f_{A_i}^h \succ g \; and \; f \succ g_{A_i}^h \; \forall i$

Savage's Theorem

Assume that X is finite. Then $\succsim$ satisfies P1–P6 if and only if there exist

- a non-atomic finitely additive probability measure μ on S $(= (S, 2^S))$
- and a non-constant ("utility:) function $u : X \to \mathbb{R}$

such that, for every $f, g \in F$:

$$f \succsim g \text{ iff } \int_S u(f(s))\, d\mu(s) \geq \int_S u(g(s))\, d\mu(s)$$

- Furthermore, μ is unique, and u is unique up to positive affine transformations.

And yet...

- If it’s so rational, why isn’t it objective?

  Isn’t it troublesome that we can’t convince others of our subjective beliefs?

  Are we do certain that we’re right?

- Are all Arbodytes Cyclophines?

  (Assume you never heard the terms before)

Are all Arbodytes Cyclophines?

- How about 50%-50%?
  - But then what about "All Cyclophines are Arbodytes"? What about the sets being disjoint, or logically independent?
  - What about "All Meta-Arbodytes are pseudo-Cyclophines"?
- (At least) I feel like saying "I don't have the foggiest idea!"
- But the Bayesian approach doesn't allow you to say this
  - How much of an idea don't you have, it asks? 74%? 73%
- The Bayesian approach is good at representing knowledge, poor at representing ignorance

How Can We Reconcile These?

- A key is the interpretation of “a state”
- Pretty modest in de Finetti
- Mixed in Savage
- But then came
    - Harsanyi: going back before we were born (“types”)
    - Aumann: considering all knowledge partitions
    - Coping with Newcomb: describe also causal relationships
    - Learning from Monty Hall: describe also the way information is imparted

→ Theoretical problems for a behavioral derivation

Disagreement among scientists – Climate Change

24 A. Millner et al.

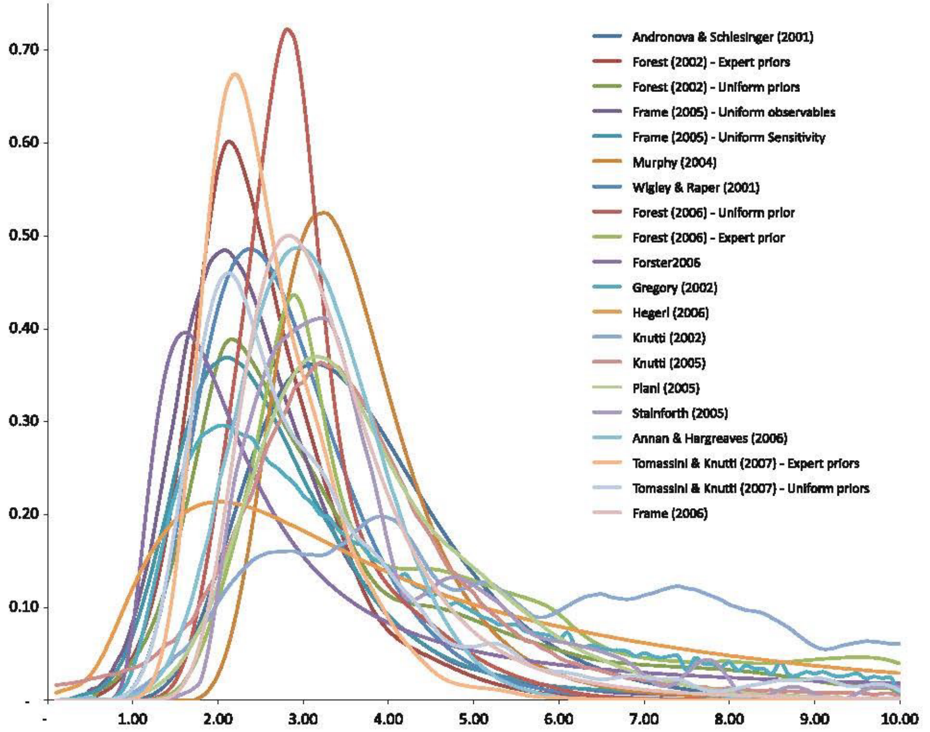
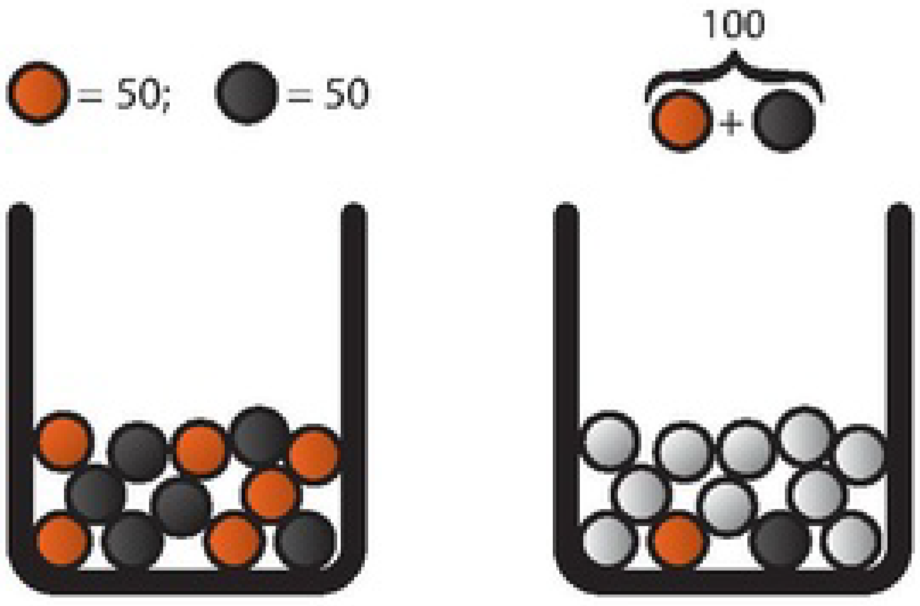


Fig. 1 Estimated probability density functions for climate sensitivity from a variety of published studies, collated by Meinshausen et al. (2009)

Disagreement among scientists – Health Risks

	John	Lisa
Mayo Clinic	25%	11%
National Cholesterol Education Program	27%	21%
American Heart Association	25%	11%
Medical College of Wisconsin	53%	27%
University of Maryland Heart Center	50%	27%

Daniel Ellsberg



Daniel Ellsberg (1931-2023)

- Would you prefer to bet on the known or unknown?
- Similar examples by Keynes (1921)

Also

- David Schmeidler found Savage's result counter-intuitive
- If I pull out a fair and well-tested coin from my pocket, I know that the probabilities are 50%-50%
- If you pull out a coin out of yours, I have no idea what the probabilities are

David Schmeidler (1939-2022)

Alternatives to the Bayesian Approach

- Schmeidler (1989): non-additive probabilities (capacities)
- Integration by Choquet's integral
- Maxmin EU: there exists a set of probabilities C such that

$$V(f) = min_{p \in C} \int_S u(f)\, dp$$

Other Multiple-Priors Models

- Nau, Klibanoff-Marinacci-Mukerji: "smooth preferences"

$$\varphi : \mathbb{R} \to \mathbb{R}$$

$$V(f) = \int_{\Delta(S)} \varphi\left(\int_S u(f)\, dp\right) d\mu$$

- Maccheroni-Marinacci-Rustichini: "variational preferences"

$$c: \Delta(S) \to \mathbb{R}_+$$

$$V(f) = min_{p \in \Delta(S)} \left[\int_S u(f)\, dp + c(p)\right]$$

Incomplete Preferences

- Bewley:

$$f \succ g$$
$$iff$$
$$\forall p \in C$$
$$\int_S u(f)\, dp > \int_S u(g)\, dp$$

- Fits the “objective rationality” notion

- Can be combined with the maxmin criterion as “subjective rationality”